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Insurance Risk

Environmental, Social and Governance Risks

Dear Readers,

Welcome to the Spring Edition of Argo Magazine, a publication that continues to explore the evolving landscape of financial regulation, modelling methodologies, and sustainable finance.

This issue brings together contributions spanning Credit Risk, Insurance, and Environmental, Social and Governance Risks, offering a comprehensive perspective on the challenges and opportunities shaping todays financial industry. Each article provides practical insights supported by rigorous quantitative frameworks, reflecting the ongoing transformation of risk management and regulatory practices.

*We open this edition with the Credit Risk section. The first contribution, **"An Interpretable Machine Learning Framework for Credit Risk Satellite Models"** by A. Mauri et al., introduces an innovative approach to developing Credit Risk Satellite Models using interpretable machine learning techniques. Leveraging the Explainable Boosting Machine (EBM), the authors aim to enhance predictive power and flexibility while maintaining the governance and transparency expected under regulatory standards. Through an empirical application based on Bank of Italy data, the paper compares ML-based methodologies with traditional econometric models across several dimensionsincluding predictive accuracy, robustness, interpretability, and operational feasibilityoffering a nuanced view of their performance under varying macroeconomic scenarios.*

*The section continues with **"Revenue Based Finance: an Evaluation Framework"** by A. Castagna, which presents a structured framework for evaluating revenue-based contracts with different contractual clauses. Through a practical application, the author illustrates the frameworks usefulness for both issuers and investors, highlighting its relevance for evolving credit strategies.*

*In the Insurance section, we feature **"Multilevel Monte Carlo for Solvency II SCR: Efficient Nested Simulation"** by G. Mori et al., a paper that addresses one of the key computational challenges in insurance modelling: the calculation of Solvency Capital Requirements using nested Monte*

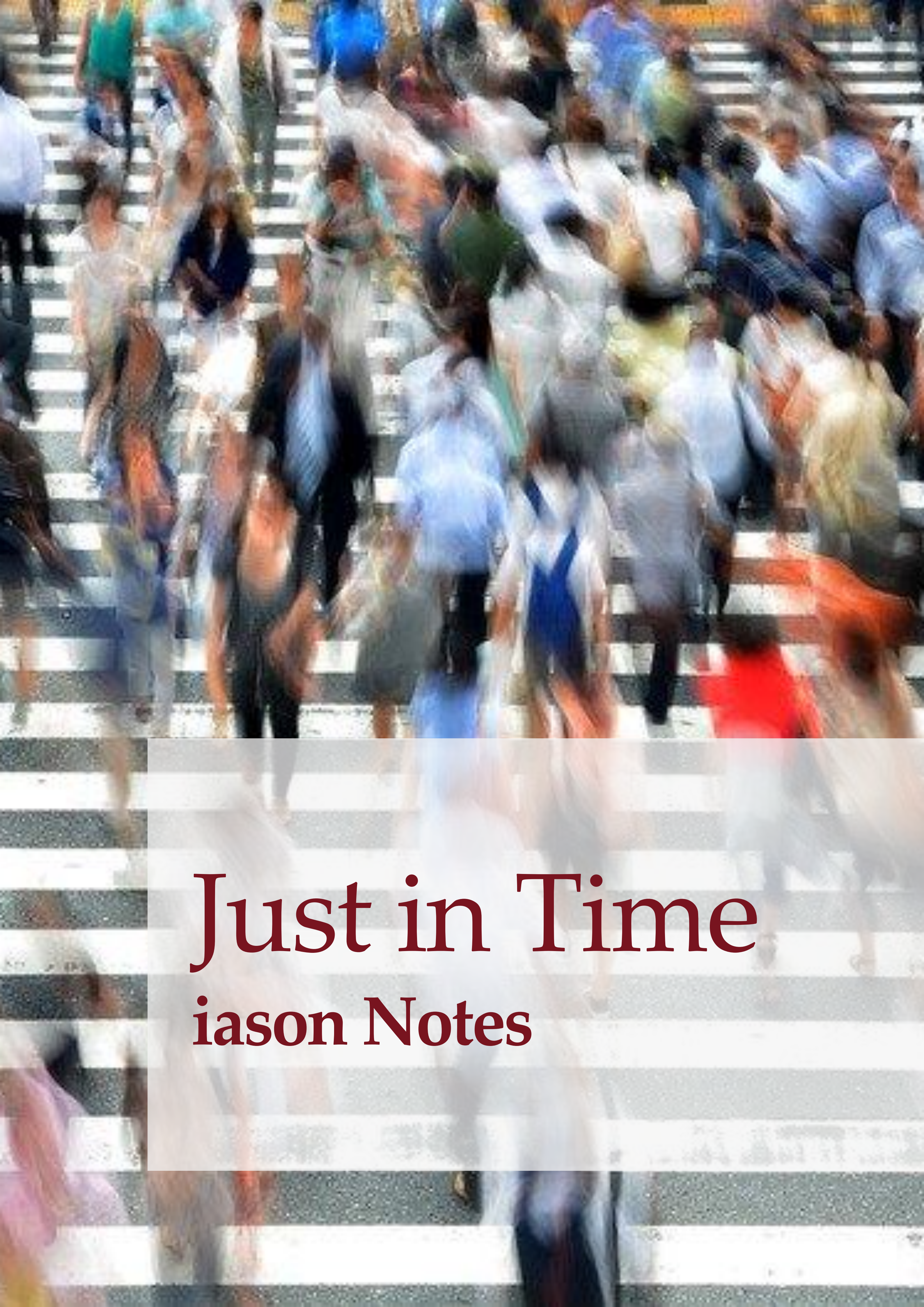
Carlo simulations. Recognising that traditional proxies may introduce model risk and require complex governance, the authors propose the Multilevel Monte Carlo method as an efficiency-enhancing alternative that preserves the nested structure without relying on proxy functions. By employing a hierarchy of simulations - from coarse to increasingly refined - they demonstrate how the method reduces variance and computational burden while maintaining accuracy. The authors further introduce an adaptive extension that concentrates computational effort on tail-risk scenarios, delivering additional improvements in efficiency and offering a risk-preserving, validation-friendly solution aligned with Solvency II expectations.

We close this edition with the Environmental, Social and Governance Risks section. The contribution ***"SFDR 2.0 and EU Taxonomy in Asset Management Field"*** *by L. Corna et al., examines the European regulatory architecture governing sustainable finance, focusing on its implications for asset management firms. The authors provide an integrated overview of the SFDR, EU Taxonomy, and CSRD, three pillars that together shape transparency obligations, the classification of environmentally sustainable activities, and corporate sustainability reporting. Particular attention is devoted to the proposed revision known as SFDR 2.0, which seeks to refine product categorisation and streamline disclosure requirements. This reform marks a significant step toward a more consistent and comparable sustainability framework, aiming to reduce greenwashing risks. For the asset management industry, the evolution of the regulatory regime presents both challenges and strategic opportunities, especially in relation to portfolio construction and ESG data governance. The authors conclude that firms capable of credibly integrating sustainability into their investment processes will be best positioned to navigate this changing environment.*

We hope that the articles featured in this Spring Edition will inspire reflection and support practitioners in navigating the ongoing transformation of financial regulation, risk modelling, and sustainable investment practices. As always, we invite you to engage with these topics, share your perspectives, and continue the dialogue with us.

We wish you an enjoyable reading.

Antonio Castagna
Luca Olivo



Just in Time

iason Notes

EBA - Discussion Paper on the Simplification and Assessment of the Credit Risk

The EBA has launched a public consultation on its Discussion Paper on the simplification and assessment of the EU credit risk framework, open until 10 May 2026. The initiative seeks to enhance simplicity, efficiency, and consistency across the framework, while preserving risk sensitivity and alignment with Basel III standards.

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Date March 2026

Draft guidelines on Credit Conversion Factor estimation

The draft Guidelines specify the methodology for estimating and applying Credit Conversion Factors (CCF) within the IRB approach, continuing the IRB repair programme and ensuring consistency with PD and LGD modelling.

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Date March 2026

SFDR Legislative Proposal: Key Changes at a Glance

Since March 2021, the Sustainable Finance Disclosures Regulation (SFDR) has outlined the requirements for financial market participants to disclose sustainability-related information to investors, with the objective of enabling these market participants to make informed decisions regarding their investments.

read more

Date February 2026

ISDA: Climate Risk Scenario Analysis for the Trading Book Phase 4 - NGFS Short-Term Scenarios

The ISDA paper addresses a critical gap in climate risk analysis for financial institutions by focusing on short-term climate scenarios (three-to-five-year horizons) and their applicability to the trading book.

While most existing climate scenarios emphasize long-term horizons (often 30 years or more), supervisory expectations and market risk management frameworks increasingly require near-term, market-consistent shocks that can be integrated into trading book stress testing, capital planning and risk governance.

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Date February 2026

Mapping Climate-Related Metrics in the Financial Sector

The OECD paper examines the state of climate-related metric disclosures in the financial sector, with a particular focus on greenhouse gas (GHG) emissions, emission reduction targets, and net-zero commitments.

It identifies persistent data gaps, methodological challenges, and inconsistencies that limit the transparency, comparability, and credibility of climate-related disclosures by financial institutions.

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Date March 2026

Hydrogeological and Credit Risk: The Italian Firms' Physical Risk-adjusted Probability of Default

The paper (1) analyses the impact of climate-related physical risks on the one-year probability of default (PD) of Italian non-financial firms, with a specific focus on hydrogeological (HG) risks, namely floods and landslides. These risks are particularly relevant in Italy due to the country's geomorphological characteristics and increasing frequency of extreme weather events.

read more

Date March 2026

Banking on Assumptions? How Banks Model Deposit Maturities

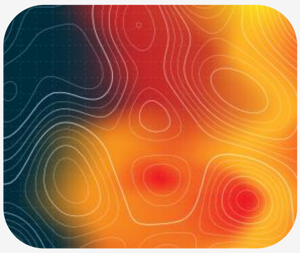
The paper investigates euro-area banks estimates on the effective maturity of non-maturing deposits (NMDs), which technically have no fixed term but often stay with banks for years. Banks rely on historical customer behavior to assign these deposits to maturity buckets rather than treating them all as having zero maturity.

Results show wide variation: roughly 20% of deposits are considered to have zero maturity, while about 10% are assumed to last more than seven years, reflecting the perceived "stickiness" of deposits.

read more

Date February 2026

EBA IRRBB Heatmap Implementation - Second Phase: Medium/Long Term Action Plan



The European Banking Authority (EBA) has published a second report on the IRRBB Heatmap, setting out medium and long-term guidance for its implementation action plan. As for the first phase, it contains observations and recommendations for Institutions and Supervisors, including some tools for Supervisors to support them in the assessment of Interest Rate Risk arising in the Banking Book (IRRBB) on several dimensions.

read more

Date March 2026

GenAI Model Risk Management and Governance in Financial Services - From Principles to Practice

Generative Artificial Intelligence (GenAI) is rapidly being adopted by Financial Institutions to support analytical, operational, and decision-support activities. While these systems deliver significant efficiency gains, they introduce new and material sources of model risk that challenge traditional Model Risk Management (MRM) frameworks - namely the U.S. Federal Reserve's SR 11-7 and the UK PRA's SS1/23.

read more

Date February 2026

Model Risk Management of GenAI Workflows

The paper examines how generative artificial intelligence (GenAI) fundamentally expands the model risk surface in financial institutions and challenges established model risk management (MRM) frameworks. While GenAI offers significant efficiency and productivity gains, it introduces novel failure modes, including open-ended outputs, composable architectures, reliance on heuristic design choices, vendor opacity, and continuous change without formal redevelopment.

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Date February 2026

Algorithmic Trading: EU Regulatory Framework and Compliance Validation Playbook

Algorithmic trading has become a central component of modern financial markets, enhancing automation, execution efficiency, and advanced risk-management capabilities. Its use, however, requires robust organisational structures and rigorous process controls, given the speed and complexity of interactions with market microstructure.

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Date February 2026

iason Weekly Insights

Regulatory/Supervisory Pills

Among iason's various publications we also find the iason Pills.

With these daily Pills, iason aims to offer a summary on information, mostly, of the main regulatory and supervisory news in the banking and finance sector on both Pillar I and Pillar II risks of the Basel framework. The main purpose of these publications is to give the reader an effective, timely and brief overview of the main topics of the moment.

The author of the Iason Pills is Dario Esposito.
read more

Market View

Among iason's weekly insight you can also find the iason Market View, a weekly update on financial market by Sergio Grasso.

The author, with almost three decades of investment experience, presents an accurate analysis of market fluctuations of the week, giving a critical view of observed phenomenos and suggesting interesting correlations with the main world events.
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An Interpretable Machine Learning Framework for Credit Risk Satellite Models

About the Authors

Andrea Mauri:
Senior Manager
Manager with 20 years of overall working experience, PhD in theoretical physics and 10+ years spent in academic research and teaching. In 2019 he moved to Risk Management with special focus on Credit Risk area. Currently accountable for Credit Risk project activities for major bank institutions, with main focus on implementation and execution of EBA, Climate and ICAAP stress test exercises, PD and LGD satellite models estimation, development of Credit Risk forecasting tools. He is also co-head of iason Credit Risk Competence Center, in charge of the technical selection, internal training and project allocation of companys Credit Risk Analysts.

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Credit Risk Quantitative Analyst
After earning a masters degree in economics and finance from the University of Milano-Bicocca, he joined Iason in November 2025 as a Credit Risk Quantitative Analyst. He is currently working on projects for one of the major European banks where he mainly carries out activities relating to the development and improvement of data models to support regulatory exercises.

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She holds a Master degree in Economics and Finance. She developed a deep knowledge of econometric models used in Finance. She is specialized in Data Science and has contributed to innovative projects including the development of a Generative AI agentic system and the estimation of satellite models for the EBA stress test for major banking institutions.

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Credit Risk Quantitative Consultant
With his MSc degree in Statistics he developed a deep knowledge of the mathematical theory behind most of the econometric models used in Finance. He is currently involved, as a Credit Risk Quant, on ICAAP and Stress Test exercises at one of the major Italian banks.

This paper proposes an innovative framework to develop Credit Risk Satellite Models, based on interpretable Machine Learning techniquesspecifically the Explainable Boosting Machine (EBM)to enhance flexibility and predictive performance while preserving governance and transparency requirements. The proposed methodology is compared with traditional econometric models in terms of predictive accuracy, robustness, interpretability, and operational feasibility, using an empirical application based on Bank of Italy data and evaluating model performance under different macroeconomic scenarios.

Satellite models represent a key statistical tool within modern credit risk management frameworks. They are extensively employed to translate macroeconomic scenarios into risk parameter projections, particularly in the context of regulatory and internal stress testing exercises, as well as within ICAAP and ILAAP processes. Moreover, satellite models play a central role in embedding forward-looking information into credit risk parameters under IFRS 9, ensuring consistency between macroeconomic expectations and the measurement of expected credit losses.

Despite their strategic relevance, industry practice still largely relies on relatively standard statistical methodologiesmost notably linear Ordinary Least Squares (OLS) regressionsto capture the relationship between macroeconomic variables and credit risk metrics (e.g., PD, LGD, or transition rates). While linear models offer transparency, interpretability, and ease of governancefeatures that are particularly valued in regulatory environmentsthey also present well-known limitations. In particular, linear specifications may fail to adequately capture non-linear dynamics, structural breaks, and regime-switching behavior that often characterize macro-financial relationships, especially during periods of stress.

In addition, the model development and selection process typically combines statistical criteria with economic reasoning and expert judgment. Expert-based interventions often influence variable selection and the final choice between models with comparable statistical performance[5]. Although this hybrid approach enhances economic interpretability and governance, it may also introduce subjectivity and lead to complex, highly customized modeling frameworks.

As a consequence, the overall modeling architecture can become cumbersome and difficult to recalibrate. Recalibration exercises frequently require full re-estimation of the models, entailing significant operational effort and potentially reducing responsiveness to rapidly changing macroeconomic conditions.

To address these limitations, alternative modeling approaches have been proposed. These approaches aim to move beyond purely linear approximations and incorporate more systematic, algorithm-driven procedures for model selection. One relevant example is Bayesian Model Averaging (BMA), which explicitly accounts for model uncertainty by combining multiple candidate specifications according to their posterior probabilities.

We have previously explored this methodology in detail, proposing an algorithmic framework for developing PD [12], sectoral PD [2], and LGD satellite models [3]. This framework also leverages the introduction of specific constraints on the model space to ensure a better economic interpretability of the resulting models [2].

Building on this line of research, the present paper introduces an innovative approach based on Machine Learning (ML) techniques, the so called Explainable Boosting Machine (EBM) [9]. The proposed framework is designed to preserve key governance requirementssuch as interpretability, stability, and regulatory transparencywhile enhancing flexibility in capturing non-linearities and complex interactions among macroeconomic drivers.

We conduct a comparative analysis between the proposed EBM methodology and more traditional approaches, highlighting their respective advantages and limitations in terms of predictive performance, robustness, interpretability, and operational feasibility. After discussing the general methodological aspects, we apply the proposed framework to estimate a set of PD Satellite Models for both Retail and Corporate counterparties, with the latter further segmented by industry sector. The calibration relies on data sourced from the public statistical database of the Bank of Italy.
We provide empirical evidence on model performance, feature explainability, and sensitivity to macroeconomic drivers. In addition, we perform empirical tests by applying the framework under different macroeconomic scenarios designed to depict an adverse global economic outlook. Finally, we conclude with a set of overall considerations and propose several directions for future research.

Methodological Overview

Main Methodologies: a Quick Review

A common workhorse for PD satellite modeling is represented by linear models, that relates a transformed measure of credit risk to a set of macro-financial drivers. In the aggregate setting, the dependent variable is often a default rate or a portfolio PD, mapped to the real line via a link function such as the logit. The resulting specification typically takes the form of a regression with lagged macroeconomic variables. This class of models is widely adopted in practice due to its transparency and ease of governance, as well as its suitability for scenario translation in macro stress testing exercises [7]. Early and influential contributions linking default rates to macroeconomic factors include [14] and [13]. The General Autoregressive Distributed lag structure is defined as:

$$y_t = \alpha + \sum_{j=1}^{p} \phi_j y_{t-j} + \sum_{k=1}^{n} \sum_{i=0}^{l} \beta_k x_{k,t-i} + \epsilon_t, \tag{1}$$

where $y_t = \text{logit}(DR_t)$ and $x_{k,t-i}$ are transforms of macro variables observed at times $t-i$. In many implementations, a large set of candidate specifications of (1) is estimated typically corresponding to different subsets of macroeconomic drivers and lag structures and a single model is ultimately selected based on in-sample fit or ad hoc validation criteria. This approach is somewhat limited because the true data-generating process is unknown, and the issue is exacerbated by the relatively small sample sizes that are common in macro-financial datasets. As a consequence, model choice may become sensitive to discretionary decisions and may vary across business units or institutions, undermining comparability and potentially leading to biased risk estimates. In particular, discretionary specification selection can entail the following risks:

- **Under-provisioning risk:** selecting an overly optimistic specification may lead to systematically underestimated DR forecasts and, consequently, an under-provisioned institution.
- **Lack of comparability across similar exposures:** institutions (or different units within the same institution) with similar exposures to macroeconomic conditions may obtain materially different DR projections simply because they rely on different handpicked specifications.
- **Distorted differentiation across heterogeneous exposures:** conversely, institutions (or different units within the same institution) with different sensitivities to macroeconomic drivers may end up producing insufficiently differentiated DR projections if model selection choices inadvertently offset or mask true exposure differences.

A way to account for model specification uncertainty is provided by Bayesian Model Averaging (BMA), which formalizes the idea that the data-generating process is unknown and multiple competing model specifications may be plausible ([8]). Let $\mathbb{M} = \{M_1, \dots, M_N\}$ denote the set of potential satellite models, and define the target as $y_t := \text{logit}(PD_t)$. Under BMA, the (model-averaged) posterior predictive distribution of y_t given the data D is:

$$p(y_t \mid D) \;=\; \sum_{i=1}^{N} p(y_t \mid M_i, D)\, p(M_i \mid D), \tag{2}$$

where the posterior model probabilities are obtained via Bayes rule:

$$p(M_i \mid D) \;=\; \frac{p(D \mid M_i)\, p(M_i)}{\sum_{j=1}^{N} p(D \mid M_j)\, p(M_j)}. \tag{3}$$

Here $p(M_i)$ denotes the prior probability assigned (by the analyst) to model M_i, while $p(D \mid M_i)$ is the marginal likelihood,

$$p(D \mid M_i) = \int p(D \mid \theta_i, M_i)\, p(\theta_i \mid M_i)\, d\theta_i, \tag{4}$$

which integrates the likelihood over the model-specific parameter vector θ_i. In practice, evaluating (4) can be computationally demanding, and applications often rely on approximations (e.g. information-criterion-based weights).

An application in the credit-risk settings is the Bayesian Averaging of Classical Estimates (BACE) proposed by [4] and [5]. Consider the previous linear satellite model specification:

$$y_t = \beta_0 + \sum_{j=1}^{q} \beta_j x_{j,t} + \varepsilon_t, \tag{5}$$

where to each model M_i corresponds a different set of β with the convention that $\beta_j \neq 0$ if the variable is included in the model specification M_i, $\beta_j = 0$ otherwise. At this point the posterior estimation given the data is:

$$\mathbb{E}[y_t \mid D, x_t] = \mathbb{E}[\beta_0 \mid D] + \sum_{j=1}^{q} \mathbb{E}[\beta_j \mid D] x_{j,t}. \tag{6}$$

and by definition:

$$\mathbb{E}[\beta_j \mid D] = \sum_{i=1}^{N} \hat{\beta}_j^{(i)} p(M_i \mid D), \tag{7}$$

with $P(M_i|D)$ as in equation (3) and $\hat{\beta}_j^{(i)}$ OLS estimate of β_j under model M_i. A practical and algorithmic implementation of BACE methodology has been introduced and discussed in details for PD [12], sectoral PD [2] and LGD [3] Satellite Models.

Alternatives to the previous approaches are machine learning (ML) methods, which can approximate complex nonlinear relationships with fewer explicit functional-form assumptions than traditional parametric satellite models. In the satellite PD setting, ML can potentially improve predictive performance and capture richer macro-to-PD dynamics, including nonlinearities and interaction effects. At the same time, the adoption of generic ML models raises challenges that are particularly relevant under stress testing and model governance requirements: limited transparency, sensitivity to hyperparameter choices, and potentially unreliable extrapolation outside the historical support of the data. Following [6] the principal models applied in the credit risk space are: Boosting Machines, Random Forest, Neural Networks. Although not all of them satisfy interpretability requirements by design, this property has become an increasingly important topic in the machine learning literature (see [1]). Thus, we define interpretability as the analyst's ability to understand and predict the behavior of a model. In this paper, we focus specifically on those ML approaches that explicitly address this constraint, prioritizing transparency as a core modeling requirement alongside predictive performance. This is particularly important because a model may learn a misspecified relationship, for instance one that is inconsistent with economic theory, while still achieving satisfactory performance on the historical data. A sufficiently interpretable model should allow the analyst to detect such potential issues. In contrast to other ML methods, which are both non-linear and non-local, we consider *Explainable Boosting Machines* (EBM) [9], an interpretable boosting framework closely related to *Generalized Additive Models* (GAM) which solves the non-locality problem, thereby facilitating model interpretation.

EBM Estimation Approach: a Quick Review

The Explainable Boosting Machine is a Generalized Additive Model that uses machine learning techniques such as gradient boosting in a way that maintains model interpretability. This type of machine learning models is called glass-box models, so models that are interpretable by design, since the way it is built allows understanding of how the model responds to the variables used in the estimation and it is possible to, for each prediction, see how a feature has contributed to the final result. The EBM takes the form:

$$g(E[y]) = \beta_0 + \sum f_j(x_j), \quad (8)$$

where $g(x)$ is a link function that depends on whether the model is estimating a regression or a classification. The satellite model estimation is solved through a regression, hence $g(x)$ is the identity function. Whereas, the $\sum(f_j(x_j))$ term is the sum of contribution predicted by each feature of the model. Moreover, the EBM allows for pairwise interaction between the features [10] with this form:

$$g(E[y]) = \beta_0 + \sum f_j(x_j) + \sum f_{ij}(x_i, x_j). \quad (9)$$

However, to maintain simplicity and preserve maximum interpretability in the satellite model case we have decided not to adopot this extension.

The EBM learns a function for each feature f_j through a gradient boosting procedure; this ensemble procedure operates on each feature independently and is implemented with a low learning rate to mitigate the impact of feature order on model training. The idea behind boosting is to train many weak learners and make them into a strong learner by combining them, so the algorithm trains the predictors sequentially and each predictor tries to fit on the residual error of the previous one [6]. The pseudocode relative to the estimation procedure is reported in Algorithm 1.

The interpretability of the EBM stems from the ability to plot each f_j and actually see the function of the contribution of each feature; moreover, since the prediction is obtained by summing the effect of individual features, it is possible to understand how each variable has contributed to the prediction.

In satellite model estimation, features carry economic meaning and a positive or negative relationship with the target variable might be expected; for this reason, in the training of the model some monotone constraints were implemented, this assures an economic variable, such as *GDP*, affects the model, following the economic expectations.

Estimation Approaches		
Approach	**Strengths**	**Limitations**
Linear / GLM	• Transparent coefficients; • Strong governance fit; • Parsimonious; • Works with small samples; • Simple scenario translation and sensitivities.	• Nonlinearities/interactions require explicit engineering; • Sensitive to collinearity, lag choices, and breaks; • Specification search can induce cherry-picking risk.
BMA / BACE	• Averages across specifications; • Reduces cherry-picking; • Quantifies driver robustness (e.g., inclusion probabilities); • Predictive uncertainty includes model risk.	• Large model space 2^{q-1}; • Computational burden; • Sensitive to priors / model design; • Nonlinearities/interactions only if explicitly included.
Generic ML	• Learns nonlinearities and interactions automatically; • Often strong predictive performance; • Scales to many predictors.	• Limited transparency, harder model governance; • Hyperparameter sensitivity; • Careful TS validation needed; • Requires a dataset of significant size.
EBM	• Interpretable additive shape functions (GAM); • Captures nonlinearities per feature; • Suitable for sensitivity inspection under scenarios.	• Binning/smoothing choices affect results; • Bagging and boosting can increase computation time; • Features values outside the in-sample range are mapped to the boundary leaf in each boosting stage.

TABLE 1: Satellite PD modeling approaches: strengths and trade-offs.

Algorithm 1: EBM Estimation Procedure

Input: Data (X, y), learning rate η, cycles C, outer bags B.
Output: Shape functions $\{\bar{f}_j\}_{j=1}^{p}$ for each regressor in X.
for $b = 1$ **to** B **do**
  Draw a bootstrap sample D^b;
  Initialize: $f_j^{(b)} \leftarrow 0$ for all j;
  set $F(x)^{(0)} = \sum_j f_j^{(b)}(x_j)$;
  Initialize the Step Number: $k = 1$;
  for $c = 1$ **to** C **do**
    for $j = 1$ **to** p **do**
      `// round-robin cycle over features;`
      Create Bins for variable j;
      Fit a small regression tree (using the gradient boosting steps) leveraging the bins to variable j and obtain estimations $h_j^{(c,b)}$;
      Update $f_j^{(b)} \leftarrow f_j^{(b)} + \eta\, h_j^{(c,b)}$;
      Update $F(x)^{(k)} \leftarrow F(x)^{(k-1)} + \eta\, h_j(x_j)^{(c,b)}$;
      Update $k = k + 1$;
Aggregate: $\bar{f}_j \leftarrow \frac{1}{B}\sum_{b=1}^{B} f_j^{(b)}$ for all $j = 1, \dots, p$;

Overall, the EBM approach appears to be a promising candidate for Satellite Models estimation. Table 1 summarizes the main advantages and disadvantages of each of the aforementioned modeling approaches.

Model Calibration

In this chapter we present the calibration framework of EBM model applied to sectoral probability of default, incorporating macroeconomic stress variables. The calibration process encompasses the specification of the dependent variable, the selection and preprocessing of macroeconomic regressors, and the definition of hyperparameter settings.

Target Variable Selection

The dependent variable is the default rate (DR) segmented by economic sector and aggregated at quarterly frequency. The default rate series are derived from Banca d'Italia Statistical Database for Italian Non Financial Corporates (NFC) and Retail [15]. Strictly speaking, the time series represent decay rates, which approximate default rates well, as both measure the flow of loans transitioning into severe distress (sofferenze rettificate vs. formal default), capturing similar systemic credit deterioration patterns over time. They thus serve as a proxy for PD, reflecting Italy's system-wide response to macroeconomic changes. First, we performed a global comparison between corporates and retail counterparties. Then, we selected five NACE sectors to analyze in greater depth how the EBM framework operates from a sectoral perspective. The selected NACE sectors are:

- Manufacturing of Construction Materials (C24-25);
- Construction and Related Services (F41-43);
- Real Estate Activities (L68);
- Power, Steam and Cooling Services (D35);
- Wholesale and Retail Trade of Vehicles (G45-47).

Separate DR series are maintained for each sector and segment, resulting in seven distinct dependent variables that are independently modeled.
To facilitate the application of the EBM framework and ensure compatibility with standard econometric assumptions regarding probability bounds, raw DR estimates are subjected to a logit transformation. The logit-transformed variable becomes the actual target in the EBM estimation procedure, after which predictions are transformed back to probability space through the inverse logit (logistic) function, ensuring all model predictions remain valid probabilities bound between zero and one.

Macroeconomic Regressors Set

The model incorporates a comprehensive set of macroeconomic variables selected on the basis of both theoretical economic reasoning (i.e. default rates rise during periods of macroeconomic contraction and financial stress) and empirical evidence from the stress-testing literature, particularly ECB and EBA methodologies for credit risk modeling.
The regressor set is gathered from publicly available standard data providers (Istat, Banca d'Italia, European Central Bank) and includes the following time series: Brent crude oil price, disposable income of Italian households, FTSE MIB index, Italian Gross Domestic Product (GDP), Italian House Price Index (HPI) and Italian unemployment rate.
Each regressor is a quarterly time series and it is considered with 0, 1, 2, 3 and 4 lags. Subsequently, all regressors are standardized to zero mean and unit variance, following the convention $\tilde{x}_{it} = \frac{x_{it} - \bar{x}_i}{\sigma_x}$, where $\bar{x}_i$ is the mean and σ_x is the standard deviation of regressor both computed on the in-sample period. This standardization procedure prevents scale-dependent regressors from dominating the boosting algorithm's feature selection process and improves numerical stability during the iterative gradient boosting procedure.
Eventually out-of-sample macroeconomic scenarios are defined following two alternative paths: a baseline scenario reflecting consensus macroeconomic forecasts conditional on current policy settings, and an adverse scenario depicting a severe but plausible economic downturn consistent with historical financial crisis episodes. Table 2 summarizes the design of the main scenario variables.

Model Estimation Framework

Following input preprocessing, models are estimated as described in the methodological overview above. Additionally, models were fitted using the following hyperparam-

Macro Variable	Scenario	2025	2026	2027
GDP	Baseline	0.80	1.10	0.90
	Adverse	-1.60	-4.40	-1.50
HPI	Baseline	2.40	2.00	1.20
	Adverse	-2.60	-6.30	-3.70
Unemployment rate	Baseline	-7.58	0.00	0.00
	Adverse	19.70	26.58	12.00

TABLE 2: Scenario Macro Variables Projection. All variables are expressed as percentage variation.

Sector and Segments	R2
C24-25	0.801
D35	0.741
F41-43	0.803
G45-47	0.804
L68	0.801
Corporate	0.747
Retail	0.801

TABLE 3: In-Sample R^2 Performance.

eters:

- No interactions: The model excludes feature interactions, estimating only univariate partial dependence functions for each regressor.
- Monotonic constraints were applied across all segments and sectors to ensure compliance with economic theory. For example, GDP growth was consistently assigned a negative constraint, preventing higher GDP values from increasing DR predictions and enforcing intuitive relationships between economic expansion and reduced credit risk for enhanced managerial interpretability.
- Stopping criteria: $R^2_{min} = 0.8$ or boosting max cycles, $C_{max} = 100$, to guarantee adequate explanatory power and avoid overfitting, ensuring stable and generalizable estimates across sectors.

Results

In this section we present the results obtained from the in-sample and out-of-sample analyses. The first part is therefore devoted to the metrics used to assess the goodness-of-fit of the models to the data, while the second provides evidence on model performance in forecasting scenarios. For further results, please refer to the Annex.

In-sample Analysis

The primary indicator used to determine how well the models explain the variability of the observed data is the Rš. Table 3 reports the value associated with this measure.
Due to the early stopping criteria, most of the models present R^2=0.8, preventing overfitting while ensuring a good adaptability to the data. The only exceptions are represented by sector D35 and by Corporate segment, where the algorithm stops after 100 iterations, preventing an increase

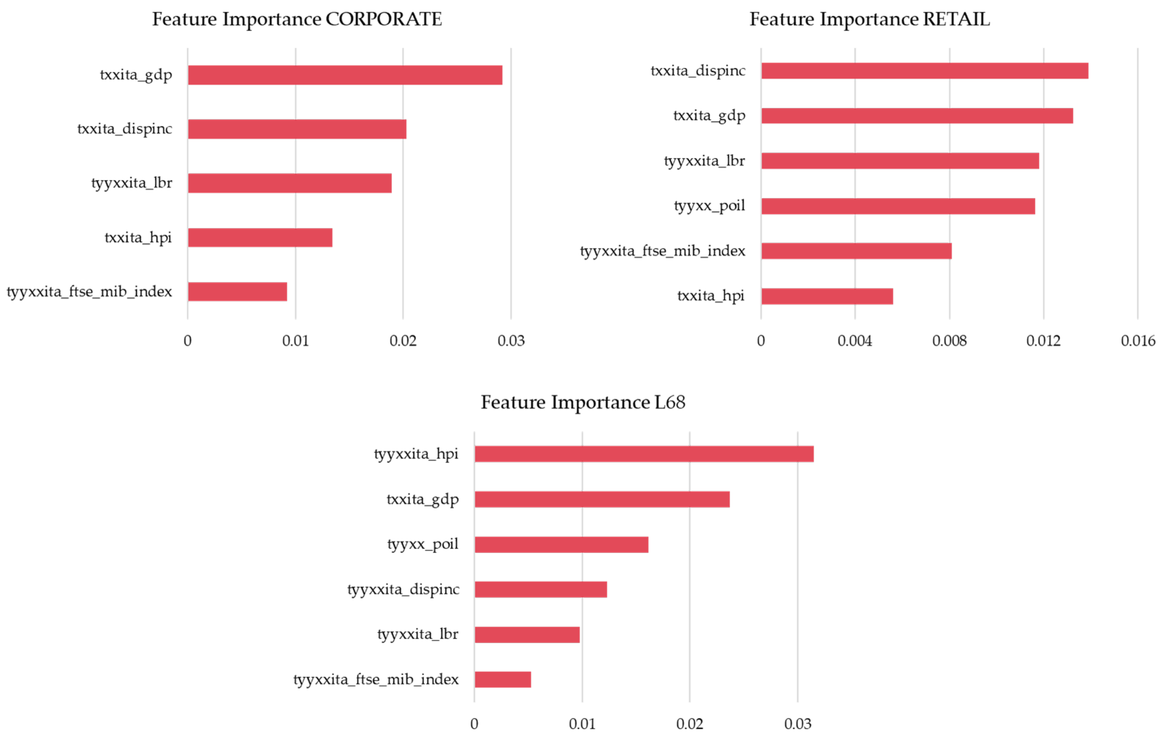

***FIGURE 1:** Feature importance for Corporate, Retail and sector L68.*

in computational time in the presence of only marginal improvements, while still ensuring a satisfactory level of fit. It is worth noting that in-sample goodness of fit should always be assessed alongside out-of-sample behavior. In this regard, we verified that, starting from the same set of regressors, the goodness-of-fit levels achieved under the OLS framework are, on average, lower than those obtained with EBM. This might be attributable to EBM's ability to capture non-linear relationships in the final model.

The EBM methodology also provides feature importance, a measure of the average contribution that each variable makes to the models in-sample predictions. For example, as shown in Figure 1, the Corporate segment is primarily driven by overall national economic performance (GDP feature importance ≈ 0.03), with households labor market conditions playing secondary roles (feature importance ≈ 0.02).

Throughout this paper, we indicate with "txx" the level of the corresponding variable and with "tyyxx" its year-on-year percentage variation. With respect to the Retail segment, greater relevance is attributed to macroeconomic variables that reflect house-

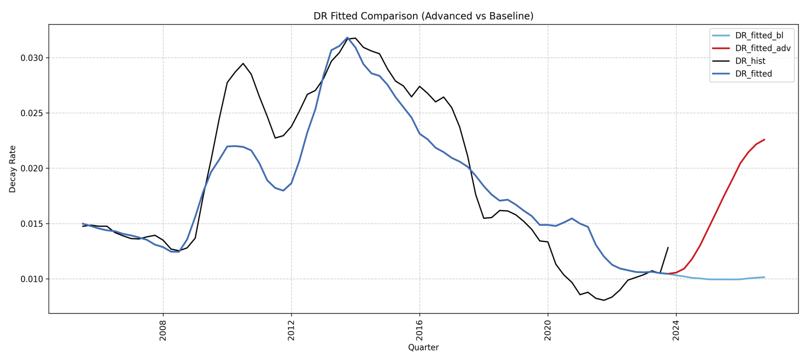

***FIGURE 2:** Historical values of DR and forecasts - Sector C24-25.*

holds labor market and financial conditions, as well as the overall performance of the national economy. Indeed, the top three positions in Figure 1 are occupied by disposable income, GDP, and the unemployment rate. Furthermore, with reference to sector L68 (Real Estate Activities), the most influential variables are the House Price Index and GDP (Figure 1).

An additional evidence is the graphical illustration of DR time series and how the models replicate the data. Figure 2 reports the series related to sector C24-25 (manufacture of basic metals and fabricated metal products). As already reported in table 3, the model appears to fits the historical data well.

Starting from the quarters following the last available observation (2023Q4), the model projections are also presented under the baseline and adverse scenarios.

Out-of-sample Analysis

After describing the in-sample results, the analysis was further extended to the out-of-sample behaviour of the models. From the graph in Figure 2, it is possible to observe that the baseline forecasts tend to settle at values consistent with those of the preceding period. Under adverse conditions, however, the projections appear inclined to reflect past periods of instability.

As an indicator of the model discriminatory power we introduce the Generalized Scenario Factor (Table 4), that quantifies how much more severe the adverse scenario is compared to the baseline one (for a more detailed description, please refer to the Annex).

The sector/segment with the largest impact is C2425, which exhibits a value of 2.23, indicating that in 2026Q4 the DR forecast under adverse conditions is 2.23 times higher than that projected under baseline conditions (0.0226 vs. 0.0101). The sector/segment least affected is Retail, with a Generalized Scenario Factor of 1.37.

To complete the set of useful indicators, a sensitivity analysis has been conducted on the most important features reported in the previous paragraph. This measure represents the average change over the forecast period in the DR, associated with an increase or decrease in a given variable, expressed both in basis points and in percentage terms.

Table 5 reports the sensitivity values for the Corporate and Retail segments, respectively. For the selected models in case of a decrease of Italian GDP of 1% in the forecast period, the DR of Italian Corporate and Retail are expected to increase by 6.08% and by 0.85% respectively, over the three years of projection. Meanwhile, in case of a increase of Italian LBR of 1% in the forecast period, the DR of Italian Corporate and Retail are expected to increase by 1.31% and by 0.89% respectively, over the three years of projection. These results indicate that Corporate segment predictions are more sensitive to GDP declines than to other variables. In contrast, Retail segment predictions show greater sensitivity to the unemployment rate, which impacts them nearly as much as equivalent GDP changes.

Since the EBM algorithm's bucketing can leave low variable variations within the same prediction level, for Retail HPI the table above shows a sensitivity of 0% over the chosen variation unit.

	Sector and Segment						
Qtime	**C24-25**	**D35**	**F41-43**	**G45-47**	**L68**	**Retail**	**Corporate**
2024Q4	1.30	1.12	1.26	1.15	1.20	1.00	1.05
2025Q4	1.91	1.35	1.65	1.43	1.51	1.18	1.50
2026Q4	2.23	1.86	1.91	1.55	1.84	1.37	1.89

TABLE 4: Generalized Scenario Factors Table.

Corporate		
Change	**Sensitivity (bps)**	**Sensitivity (%)**
1% decrease in Italian GDP	8.16	6.08%
1% decrease in Italian HPI	1.04	0.78%
1% increase in Italian LBR	1.76	1.31%

Retail		
Change	**Sensitivity (bps)**	**Sensitivity (%)**
1% decrease in Italian GDP	0.62	0.85%
1% decrease in Italian HPI	0	0.00%
1% increase in Italian LBR	0.65	0.89%

TABLE 5: Sensitivity in Corporate and Retail segments.

Conclusions

In light of the analyses and results discussed in the previous sections, we draw conclusions regarding (i) the empirical performance of the proposed framework, (ii) the economic interpretability of the identified drivers, and (iii) potential directions for further development.

From a predictive and scenario-consistency perspective, the model performs well across all the provided time series. In the in-sample analysis, the EBM closely fits the historical DR series for every segment and sector considered. Importantly, this fit remains credible when moving beyond the estimation window. Indeed, in the out-of-sample evaluation, there is no evidence of overfitting: projections follow the scenario design and preserve coherence between the baseline and adverse paths. Moreover, DR projected levels remains consistent with the historical range. Overall, these results indicate an appropriate trade-off between explanatory power (as measured by R^2) and out-of-sample robustness. This positive trade-off is further supported when also considering the computational effort. Although EBM is inherently more computationally intensive than a single OLS estimation, its overall computational burden proves to be lower when considered in the context of a full model selection workflow. In a standard OLS framework, achieving a reliable specification typically requires estimating a large number of candidate models followed by a post-estimation selection step. With EBM, this entire process is bypassed, as the algorithm directly delivers a single, data-driven specification. Moreover, the training time of EBM can be substantially reduced through appropriate hyperparameter tuning for instance, by decreasing the number of outer bags making it well-suited for the limited sample sizes typical of stress testing exercises.

Building on this performance evidence, the model outputs also provide economically meaningful insights into the drivers of credit risk dynamics. In the comparison between corporate and retail segments, EBM confirms intuitive relationships: GDP emerges as the main explanatory feature for corporate DR, while labor market indicators and disposable income play a predominant role for retail counterparties. This segment-level consistency strengthens confidence that the model is not only accurate, but also behaviorally plausible.

The same interpretative value extends naturally to the sectoral dimension, where EBM is able to distinguish heterogeneous sensitivities across the five sectors analyzed. In particular, the Manufacturing of Construction Materials sector (C24C25) appears to be the most affected under the adverse scenario, whereas the Wholesale and Retail Trade of Vehicles sector (G45G47) is the least sensitive. Coherently, these differences are reflected in the feature-importance evidence, which highlights sector-specific leading macroeconomic variables and suggests that sectoral PD dynamics are driven by distinct macroeconomic channels.

These empirical and economic findings motivate the methodological conclusion that EBM represents a particularly effective compromise between model flexibility and interpretability. The approach can capture non-linear relationships between macroeconomic variables and DR while retaining a level of transparency comparable to linear specifications. Additionally, the availability of monotonic constraints is especially valuable in stress testing contexts, where sign-consistency and economic rationale are pivotal.

Finally, the results suggest several extensions that could further strengthen the framework and clarify its applicability boundaries. First, testing additional scenarios and expanding the set of sectors/segments would enable more robust conclusions on model stability under alternative macro-financial conditions. Second, moving from quarterly to monthly DR series could improve learning capacity by increasing the number of observations, which is often beneficial for machine learning methods. Third, other extensions incorporating pairwise terms while preserving both the explainability and the ability to impose constraints on the functional form should be investigated. Lastly, extending the approach to Loss Given Default (LGD) modeling, typically harder to explain using macroeconomic variables alone, would provide a meaningful benchmark to evaluate whether the interpretability advantages observed for PD also hold for other risk parameters.

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Annex

In this annex, for illustrative purpose, we present a more detailed set of results for industrial sector C24-25 (Manufacturing of Construction Materials). We report the DR forecasts for the different scenarios at first, the second section examines the sensitivity of the forecasted DR with respect to the GDP macro variable, and the third section presents the feature importance analysis. Some of the presented output were obtained using the InterpretML package [11].

Final Results Summary

In this part of the annex a comparison between the Baseline scenario and the Adverse scenario is reported. Table 6 presents the predicted DR values (for Baseline and Adverse), the predicted delta DR with respect to the last observation in the historical set (2023q4), and the Generalized Scenarios Factor, calculated as $DR_{adv,t}/DR_{bl,t}$.

Sensitivity

In this section we report the plots of the sensitivity to GDP of the DR. The sensitivity is calculated by taking the last predicted DR in the in sample (2023q4) and seeing how it changes by keeping all the regressors fixed except the current GDP level (2023q4) normalized to 1, which is multiplied by a factor changing between -15% and $+15\%$ by steps of size 1%. The results for sector C24-25 are presented in Figure 7.

Qtime	DR fitted baseline	DR fitted adverese	Delta DR fitted baseline	Delta DR fitted adverse	Generalized Scenarios Factor
2023Q4	0.010	0.010			1
2024Q4	0.010	0.013	-0.040	0.248	1.299
2025Q4	0.010	0.018	-0.049	0.815	1.908
2026Q4	0.010	0.023	-0.030	1.162	2.228

TABLE 6: Comparison Baseline vs Adverse predictions, Sector C24_25.

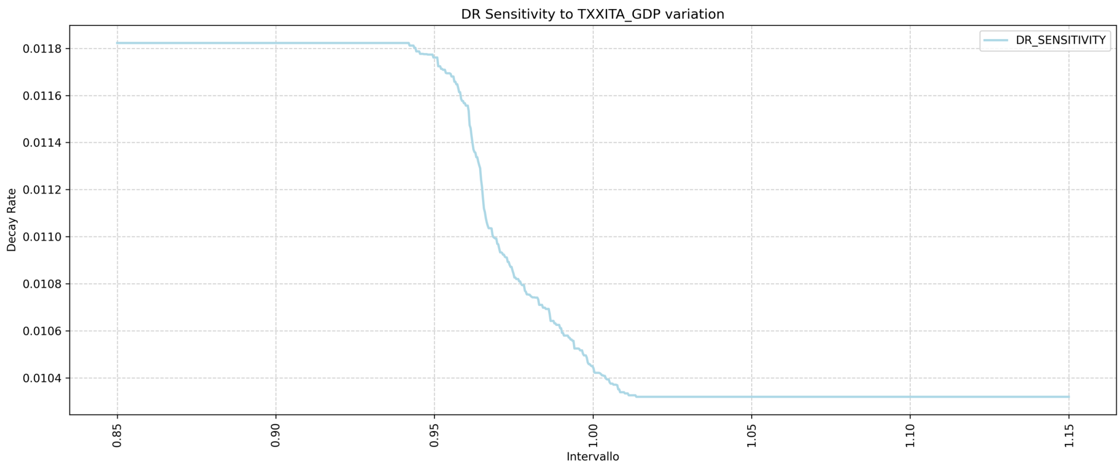

FIGURE 3: GDP sensitivity for sector C24_25.

Change	Sensitivity (bps)	Sensitivity (%)
1% decrease in Italian GDP	0.572	0.570%
1% decrease in Italian HPI	4.105	4.090%
1% decrease in Italian DISPINC	0.000	0.000%
1% increase in Italian LBR	0.005	0.005%
1% decrease in FTSE MIB INDEX	0.000	0.000%

TABLE 7: Sensitivity in sector C24_25.

Furthermore, sensitivity analysis for different explanatory variables is reported in the table below. As we mentioned in the previous sections of this research, this measure represents the average change over the forecast period in the DR, associated with an increase or decrease in a given variable, expressed both in basis points and in percentage terms. As it can be noted, in the Table 7 above we report also the result for Disposable Income, which exhibits 0% sensitivity over one-percent variation, because its scenario projections fall outside observed historical boundaries. Similarly, the FTSE MIB shows no sensitivity over the chosen variation unit, as the EBM algorithm's bucketing can leave low variable variations within the same prediction level.

Feature Importance

In this final part we report the Feature Importance graphs, calculated as $\frac{1}{n}\sum_{i=1}^{n}|f_j(x_{ij})|$. The Second graph represent the mean importance grouping by regressors name.

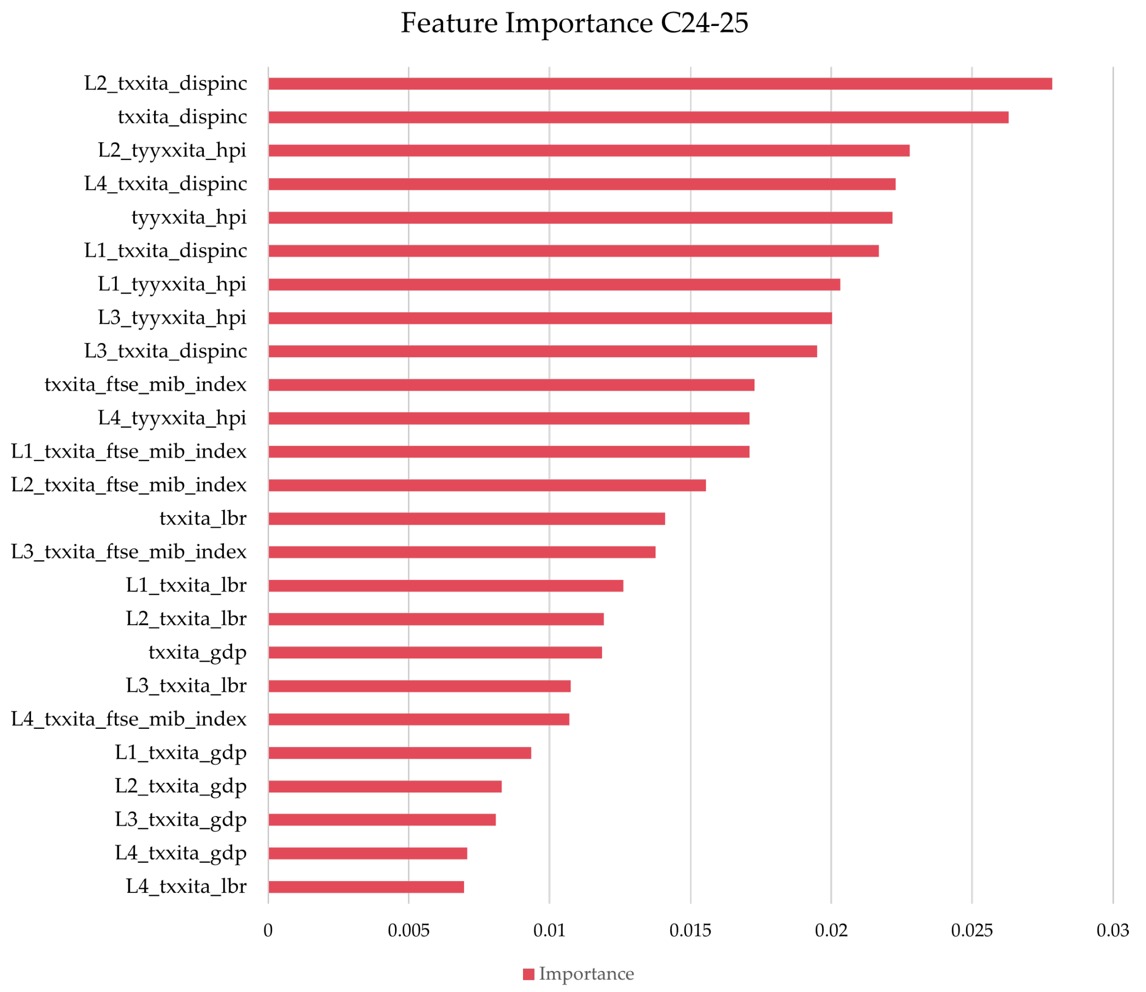

FIGURE 4: Feature Importance for sector C24_25.

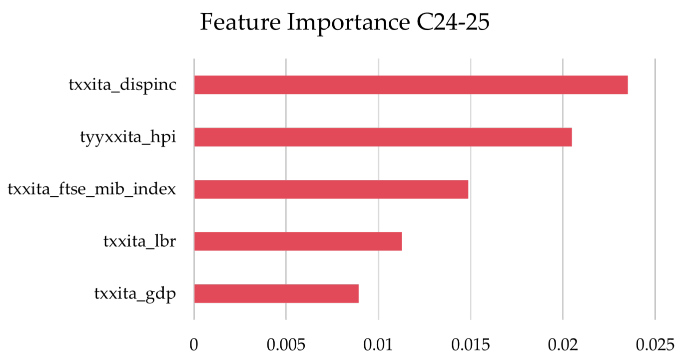

FIGURE 5: Mean Feature Importance for sector C24_25.

Revenue Based Finance: an Evaluation Framework

About the Author

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Founder and Managing Partner
Antonio Castagna is currently managing partner and founder of the consulting company iason. He previously was in Banca IMI, Milan, from the 1999 to 2006: there, he first worked as a market maker of cap/floor's and swaptions; then he set up the FX options desk and ran the book of plain vanilla and exotic options on the major currencies, being also responsible for the entire FX volatility trading. He started his carrier in the investment banking in the 1997 in IMI Bank, in Luxemborug, as a financial analyst in the Risk Control Department. He graduated in Finance at LUISS University in Rome in 1995, with a thesis on American options and the numerical procedures for their valuation. He wrote papers on different topics, including credit risk, derivative pricing, collateral management, managing of exotic options risks and volatility smiles. He is also author of the books "FX options and smile risk" and "Measuring and Managing Liquidity Risk", both published by Wiley.

We present a framework to evaluate Revenue-Based contracts, with different clauses. The framework is used in a practical application.

Revenue-Based Financing (RBF) has recently experienced remarkable growth among private companies. In a nutshell, RBF is an alternative funding model in which investors provide capital to a business in exchange for a fixed percentage of the business's future gross revenues. There are both advantages and disadvantages for debtor companies when they access this form of financing, and we will examine them below. We will also show that, under the umbrella of RBF, several funding schemes can be grouped together, even though we will focus on the pricing and contract evaluation of arrangements strictly linked to cash flows derived from the company's revenues.

More specifically, we will outline a framework for evaluating RBF contracts that accounts for the main features of typical agreements and the risk factors affecting their value. In some cases, this framework can be easily adapted and extended to different types of agreements, as we will see below.

An Overview of Revenue-Based Finance

Revenue-Based Financing is a form of alternative funding that may be preferable to more traditional financing contracts for several reasons. Firstly, it is non-dilutive, as it does not involve equity shares; secondly, it allows for flexible repayments, since payments are proportional to the revenues generated by the business; thirdly, unlike most traditional loans, no collateral is required, whether in the form of personal guarantees or assets.

To these advantages, one can also add the typically faster access to funding offered by RBF compared to loans granted through the banking channel, and the alignment of interests between the debtor company and the financing investor, since the loan's repayment depends on actual future business revenues.

On the other hand, RBF also has some disadvantages: the cost of capital is typically higher than traditional funding; it is not ideal for seasonal or volatile businesses, as investors prefer constant and stable repayments; the impact on available cash can be significant, especially when repayment is based on monthly revenues; finally, the size of funding is limited and usually smaller than equity rounds or traditional loans.

Different funding arrangements can fall under the definition of "Revenue-Based Finance". In Table 6, we provide a quick summary of the types of contracts and their main repayment terms. The first three-Merchant Cash Advances, Royalty-Based Financing, and SaaS (Software-as-a-Service) Financing-are the only ones strictly aligned with the definition provided above: repayment derives from the stream of revenues generated by the business. These revenues can originate, respectively, from sales, from fees related to royalties, or from fees for the use of software products.

The last two types of contracts, Invoice Financing and Inventory Financing, are indirectly related to the revenues of debtor companies. Invoice Financing is currently a very common funding product for working capital, and many web-based platforms provide the infrastructure for smooth processing of all phases of the negotiation between companies and financing investors. The basic features of this financing arrangement are presented and analysed in Castagna [1]. Inventory Financing, although still related to the company's sales, is a form of financing collateralised by the inventory of products: it is suitable only for businesses selling physical goods.

In this work, we will focus on the first three types of contracts. The evaluation frame-

	Merchant Cash Advances	Royalty-based Financing	SaaS Financing	Invoice Factoring	Inventory Financing
Ideal for:	Businesses with steady credit card sales.	Technology, entertainment or product development businesses.	Software companies with predictable subscription revenue.	B2B businesses with unpaid invoices needing immediate cash flow.	Retailers, wholesalers or manufacturers that can use inventory as collateral.
Main contract terms:	Typically up to one year.	Flexible, tied to specific product or project revenue.	3 to 5 years.	70% to 90% of invoice value upfront.	Up to 80% of inventory value.
Repayment terms:	10% to 20% of daily sales.	1% to 10% of revenue from the project or product.	1% to 10% of Monthly Recurring Revenues.	Based on invoice due dates, usually 30 to 120 days.	Typically 6 to 36 months (paid in monthly installments or when inventory is sold).
Cost:	Factor rates of 1.1x to 1.5x times the amount borrowed.	1.5x to 2.5x repayment cap, based on agreed-upon royalties.	Repayment caps typically range from 1.5x to 2.5x.	Fees range from 1% to 5% of invoice value.	May be fees for inventory appraisal, loan origination, prepayment, etc.

FIGURE 6: *Types of revenue-based contracts and their main features.*

work outlined below can be applied to all of them, their differences being mainly terminological. Apart from the typical (implicit) duration of each of the three contracts, the mechanics of the funding arrangement can be summarized in the following points:

- The company receives upfront funding from the investor.
- The business agrees to pay a fixed percentage (typically 5-15%) of recurring revenues for a given period; for example, monthly recurring revenues (MRR) or annual recurring revenues (ARR).
- Payments continue until the investor receives a multiple of the original investment (*e.g.*, 1.4x); this amount is usually termed the *repayment cap*.

We can delve into each of these points to analyse some usual contract terms.

Lent Amount

Generally, the debtor company receives a given sum of funds at the inception of the contract; the repayment amount is typically expressed as a multiple of the lent amount. Sometimes, the contract terms provide for a notional amount from which an upfront fee is deducted. This net amount is what is actually lent to the debtor company, which will then make payments in the future based on business revenues until the notional amount is fully repaid. The difference between the "multiple" and "upfront fee" methods of indicating the lent amount and the repayment cap is merely semantic, with no impact on the financial evaluation when the two alternatives are brought to comparable actual lent amounts.

Expiration Date

As can be easily seen from the points above, there is no fixed end date, and the contract duration depends on revenue performance: higher-than-expected recurring revenues will accelerate repayment and hence shorten the expected duration, while

the opposite will occur with lower-than-expected revenues.
In some cases, to protect the lender from an unreasonably long duration if revenues permanently collapse, a term date can be set in the contract. If this is the case, upon the term date all the outstanding lent amount is repaid to the lender, regardless of the percentage that this amount represents with respect to the revenues of the last reference period. When a term date is provided in the contract, the maximum duration of the contract is known, whereas the amounts on each repayment date still depend on business revenues-except for the last one.

Types of Revenues

An RBF contract typically defines "revenues" as the cash receipts generated by the business activity during the reference period: a dedicated bank account that can be monitored by the lender is set up by the borrower, and all cash receipts are paid into it.
A much less common variation, for companies not selling to retail clients but only to other businesses, is the definition of revenues as all invoices issued in a given period, regardless of their actual payment by the client. In this case, outstanding invoices issued during the reference period can be transferred to the lender, who will receive payment directly from the clients. The transfer of invoices can be with or without recourse, depending on whether the borrower remains ultimately liable for any missing payments for any reason.
The cash flows needed for the repayment are those generated by the overall business of the debtor company, but in some cases they can originate from specific receivables that are transferred to the lender. As an example, in royalty-based and SaaS financing, the underlying contracts can be transferred to the lender so that the cash flows paid by the counterparties of these contracts are (possibly fractionally) used to repay the loan. In addition, invoices issued by the debtor company when the loan starts-or to be issued at future dates-can be transferred to the lender, and the cash flows arising when they are paid by the debtor's clients are directly used to repay the debt.

Repayment of the Lent Amount

The repayment of the amount by the debtor is made at the end of each reference period and consists of a share, or rate, of the business revenues or receipts from a royalty or SaaS contract (*e.g.*: 5% of monthly recurring revenues).
Sometimes, a floor can be provided in the contract, so that repayment at the end of a reference period occurs only if business revenues exceed a given amount. If this is the case, a floor may slow down the repayment of the lent sum and extend the duration of the contract, if a term date is not specified.

Default Risk

The main credit risk that the lender bears is that the borrower defaults: if the business of the debtor does not generate sufficient revenues to cover costs, the default of the debtor company becomes inevitable, and the lender will likely suffer a loss on the outstanding lent amount. This case applies to RBF contracts where business cash receipts received by the debtor company are used to repay the debt.
When revenues are collected directly by the lender through the assignment of invoices-whose payment produces the cash to repay the debt-we must also consider the default risk of the debtor company's clients, depending on the terms of the RBF contract. In more detail, we may have two cases:

- Full Recourse Loan;
- Limited Recourse Loan.

In the first case, the debtor company is liable for all missing invoice payments by its clients, so the relevant default risk to consider in evaluating an RBF contract is only that of the borrower. In the second case, the lender bears the risk of missed payments, even if no immediate loss on the outstanding capital occurs. In fact, if one or more

	Cash Advance	Cash Advance with Invoice Assignment	SaaS Financing	Royalty-based Financing
Full Recourse	- Default risk of the debtor company.	- Default risk of the debtor company.	- Default risk of the debtor company.	- Default risk of the debtor company.
Limited Recourse	N/A	- Default risk of the debtor company; - Default risk of the counterparty, relevant for the duration of the assigned invoices.	- Default risk of the debtor company.	- Default risk of the debtor company.

FIGURE 7: Relevant default risks to consider in the evaluation of different RBF contracts.

invoices are not paid, the repayment of the lent amount is smaller than expected, but it can still be completed in the future. Only if the debtor company defaults and business activity stops does the lender suffer a loss on the unpaid portion of the loan.

We stress that the RBF contract provides for the assignment of a fraction of invoices issued in the future, with client companies that may not be identified at the inception of the financing. Thus, we refer to the assignment of invoices to a generic client company: this implies that there is always a surviving generic company whose invoices are transferred to the lender, and the default of this generic client could be material only during a short period of time, *i.e.*, the time span between the assignment and the expiry of the invoices.

Consider, for example, the assignation to the lender of some invoices issued by the debtor company, at some point in the future, so that their payment by the client companies provide the funding for a repayment instalment: in this case, the client company default is relevant only between the assignation date and the expiry date of the invoices. It is quite likely that the RBF contract starts on a date long before the invoices are issued, so that lent amount can be actually seen as a payment in advance for the purchase of invoices to be issued in the future (*e.g.*: one year), that are due according to the terms of the business relationship between the debtor and client companies, (*e.g.*: one month after the issuance).

On the other hand, the default of the debtor company is always relevant, even before the issuance and sale of the account receivables, since when its default occurs, no more revenues will be produced and, consequently, no more invoices can be transferred. Because the invoices are paid by the buyer of the revenue-based claim at the inception of the contract-possibly before the invoices are issued and transferred-the default of the debtor company likely causes a larger loss.

Also in royalty- and SaaS-based contracts, the default risk of the client company should be included in the evaluation of the RBF according to the type of arrangement, which can provide for full or limited recourse to the borrower, as in the case of invoice assignment. With full recourse, the debtor company (*e.g.*, a SaaS company) bears the client's default risk: if the SaaS or royalty-paying client does not pay or terminates the contract, the borrower is obligated to repay the outstanding amount of the loan, just like a normal debt. Thus, SaaS revenues are the source of the cash flows used to repay the debt, but if they stop for any reason, there is no limitation of liability for the debtor company. In evaluating an RBF contract, the default risk that matters is only that of the debtor, since it is ultimately liable for full repayment of the debt.

With limited recourse, the risk of default of the client company falls on the lender, since repayment of the debt is limited to actual receipts from the SaaS contract. So, if the client defaults or the contract revenue stops for any other reason, repayment ends and the lender cannot recover the full lent amount. In this case, the evaluation of an RBF contract should take into account the default risk of the client company and the default risk of the debtor company if this entails termination of the royalty or SaaS contract with the client, as is usually the case.

In royalty- and SaaS-based loans, the counterparty of the underlying contract is typically identified at the start of the agreement, so we are not considering a generic client but a specific company. This means that the default risk of the debtor's client is relevant at any time during the duration of the RBF contract and not just for limited periods, as we saw before in the case of invoice transfers.

A summary of the relevant default risks for the different types of RBF contracts is shown in Table 7.

In what follows, we will focus on financing contracts strictly referring to the business revenues of the debtor company, since it is not particularly difficult to handle royalty- and SaaS-based contracts using standard approaches. In fact, the stream of receipts deriving from royalties and SaaS contracts is usually well defined in the agreements and does not require any special modeling treatment. If these receipts are stochastic, the modeling approach adopted for business revenues can be straightforwardly applied to royalties or SaaS receipts.

Modelling the Revenues of the Debtor Company and Other Relevant Risk Factors

To evaluate RBF contracts, we need a framework that considers all relevant risk factors: we must stress that most of these factors cannot be, directly or indirectly, traded on the market. As such, we have to consider real-world stochastic processes and assume an equilibrium model that includes risk premia, allowing the passage to risk-neutral processes and, hence, risk-neutral evaluation.

Credit Risk

The default of the debtor company is modeled following a reduced-form approach, *i.e.*, by directly modeling the default event as a jump process. In more detail, let $\Lambda_t = \mathbf{1}_{t \geq \tau}$ be the indicator function of the event "default" at time τ. It is a stochastic process, specifically a Poisson process. If λ is assumed to be the instantaneous arrival intensity rate, then we have:

$$\mathbf{E}[d\Lambda_t] = \lambda\, dt.$$

We assume that the default intensity follows (in the real world) as square-root mean reverting process, or a CIR process (from the Cox, Ingersoll and Ross, who used this process for the instantaneous interest rate, see [3]):

$$d\lambda(t) = \kappa_\lambda[\theta_\lambda - \lambda(t)]dt + \sigma_\lambda \sqrt{\lambda(t)}\, dZ_\lambda(t). \tag{10}$$

We define the survival probability $\mathbf{SP}(t, T)$ between time t and T as:

$$\mathbf{SP}(t, T) = \mathbf{E}\left[e^{-\int_t^T \lambda(s)ds}\right], \tag{11}$$

which admits an explicit solution in the CIR setting, as shown below.

For the evaluation of some revenue-based contracts, it is useful to introduce an additional risk factor: the probability of default of the clients of the debtor company. The receivables transferred to the lender generate the cash flows that repay the debt: when the debtor's client company goes bust, the revenue-based claim suffers a missed payout (which can be covered by the debtor if full recourse is allowed, see above).

To model this risk, in case the RBF contract provides for the assignment of invoices, we assume that the receivables are paid by a representative generic client company;

when the client company can be unequivocally identified, the default refers to this specific company.

Let $\Xi_t = \mathbf{1}_{t \geq \tau^c}$ be the indicator function of the event "default" of the client company at time τ^c. As in the case of the debtor company's default, it is a stochastic process, specifically a Poisson process, with instantaneous arrival intensity rate ξ, so that:

$$\mathbf{E}[d\Xi_t] = \xi\, dt.$$

Also in this case, the default intensity follows the square-root mean reverting process:

$$d\xi(t) = \kappa_\xi[\theta_\xi - \xi(t)]dt + \sigma_\xi\sqrt{\xi(t)}\, dZ_\xi(t). \tag{12}$$

The survival probability $\mathbf{SP}^c(t,T)$ between time t and T is:

$$\mathbf{SP}^c(t,T) = \mathbf{E}\left[e^{-\int_t^T \xi(s)ds}\right]. \tag{13}$$

Whether to include or not, and to what extent, the default risk of the client of the debtor company depends on the terms of the RBF contract, as we showed above.

Revenues

Business revenues are modelled with respect to the reference period provided for in the contract: we saw above that the repayment of the loan is made by means of a fraction of the recurring revenues generated in a given period (say, one month). So, we need to model the recurring revenues for this period of time. Let us denote them with V^* and assume they follow the process:

$$dV^*(t) = \mu V^*(t)\, dt + \sigma_V V^*(t)\, dZ_V(t). \tag{14}$$

This is a standard geometric Brownian motion widely used in finance: it is a continuous process, but in practice observed only at the end of each reference period. More specifically, given the duration of the loan contract $[0,T]$, which includes N end-of-period dates $T_1, T_2, \ldots, T_N$, the process $V^*(t)$ will be observed-and hence its value measured-on these N end-of-period dates (for example, at every month-end). It should be stressed that V^* does not represent cumulative revenues over the interval $[0,T]$: it refers only to the revenues for each reference period within the duration of the loan contract, which are fictitiously assumed to follow a continuous process, even though their value matters only at the levels attained on the N end-of-period dates.

Now define two processes: the revenue process $V^*(t)$, which ignores any effect of a default event; the process $V(t)$, which equals $V^*(t)$ before default occurs, that is $V(t) = V^*(t) \cdot \mathbf{1}_{t<\tau}$, and it drops to 0 after the default event, so that it can be written as:

$$V(t) = 0, \quad t > \tau.$$

Applying the Ito's lemma and the analogous lemma for the Poisson process, one gets the following dynamics for the process $V(t)$:

$$\begin{aligned} dV(t) =& \mu V(t)\, dt \\ &+ \sigma_V V(t)\, dZ_V(t) \\ &+ [V(\tau) - V(t)]\, d\Lambda(t). \end{aligned} \tag{15}$$

It is useful to note that, since $V(\tau) = 0$, we have that the expectation of the last term of the SDE (15) is equal to $\mathbf{E}[-V(t)d\Lambda(t)]$ and it can be written as:

$$\mathbf{E}[-V(t)d\Lambda(t)] = \mathbf{E}[-V(t)|d\Lambda(t) = 1]\cdot$$

$$\cdot\mathbf{E}[d\Lambda(t)] = -V(t)\lambda(t)\, dt,$$

so that the dynamics of the revenues is:

$$dV(t) = [\mu - \lambda(t)]V(t)\, dt + \sigma_V V(t)\, dZ_V(t). \tag{16}$$

The process $V(t)$ may be useful to include in a compact fashion the debtor's default risk in the evaluation formulae.

Interest Rate

Interest rates are modelled through the evolution of the instantaneous rate, which assumed to follow a mean-reverting square root model as in Cox, Ingersoll and Ross [3]:

$$dr(t) = \kappa_r[\theta_r - r(t)]\, dt + \sigma_r \sqrt{r(t)}\, dZ_r. \quad (17)$$

The price at time t of a zero-coupon bond $P(t, T)$ expiring at time T can be calculated as:

$$P(t, T) = \mathbf{E}\left[e^{-\int_t^T r(s)ds} \right], \quad (18)$$

which admits a closed-form solution in our setting (see below). The price of a zero-coupon bond can also be seen as the discount factor applied to future cash flows to compute their present value.

Evaluation of Revenue-Based Claims

To evaluate revenue-based claims (or, more generally, contracts), we must consider that most risk factors cannot be hedged with market instruments: no contract on revenues, nor CDS on a small company (the typical debtor), is actively traded on the market; only interest rates can be hedged through contracts traded on the market. Therefore, we cannot resort to pricing via a replicating portfolio *á la* Black&Scholes; instead, we need an equilibrium model such as the one described in Cox, Ingersoll, and Ross [2].

The PDE of Revenue-Based Claims

A claim on the revenues of a company can be mathematically defined as a function $J(V(t), r(t), \lambda(t), \xi(t), t)$ with a dynamics defined as:

$$\begin{aligned} dJ(V(t), &r(t), \lambda(t), \xi(t), t) \\ &= \mu_J(V(t), r(t), \lambda(t), t)\, dt \\ &+ \sigma_J(V(t), r(t), \lambda(t), t) dZ_J \\ &+ [J(\tau^c) - J(t)]\, d\Xi(t). \end{aligned} \quad (19)$$

The value of the claim depends on time, revenues, interest rates and the default risks of the debtor and client companies. The default of the client company directly affects the value of the claim through the term $[J(\tau^c) - J(t)], d, \Xi(t)$, whereas the default risk of the debtor company directly affects the revenues and thus indirectly the value of the claim.

By setting $J(\tau^c) = 0$ (*i.e.*: the claim is worth nil when the client company defaults), from Ito's lemma, simplifying the notation, the drift of the claim is:

$$\begin{aligned} &\frac{1}{2}\sigma_V^2 V^2 J_{VV} + \frac{1}{2}\sigma_r^2 r J_{rr} + \frac{1}{2}\sigma_\lambda^2 \lambda J_{\lambda\lambda} + \frac{1}{2}\sigma_\xi^2 \xi J_{\xi\xi} \\ &+ \mu V J_V + \kappa_r(\theta_r - r) J_r \\ &+ \kappa_\lambda(\theta_\lambda - \lambda) J_\lambda + \kappa_\xi(\theta_\xi - r) J_\xi \\ &- \lambda V J_V - \xi J + J_t - \xi J, \end{aligned} \quad (20)$$

where J_x and J_{xx} indicate, respectively, the first and second derivatives of the claim value function J with respect to the variable x (we have also slightly lightened the notation).

On the other hand, in an economy described in Cox, Ingersoll and Ross [2] and under the assumption in Cox, Ingersoll and Ross [3], no-arbitrage conditions imply that any claim whose value depends on the price of the revenues' value must have an instantaneous drift equal to:

$$\left(r + \pi_V V \frac{J_V}{J} + \pi_r r \frac{J_r}{J} + \pi_\lambda \lambda \frac{J_\lambda}{J} + \pi_\xi \xi \frac{J_\xi}{J} \right) J,$$

where π_i, $i \in \{V, r, \lambda, \xi\}$ are the market prices of risk for the three factors respectively.

By equating the drift in SDE (19), explicitly derived by means the Ito's lemma, to the drift obtained on no-arbitrage conditions, we have:

$$\frac{1}{2}\sigma_V^2 V^2 J_{VV} + \frac{1}{2}\sigma_r^2 r J_{rr} + \frac{1}{2}\sigma_\lambda^2 \lambda J_{\lambda\lambda} + \frac{1}{2}\sigma_\xi^2 \xi J_{\xi\xi} + (\mu - \pi_V) V J_V + [\kappa_r(\theta_r - r) - \pi_r r] J_r + [\kappa_\lambda(\theta_\lambda - \lambda) - \pi_\lambda \lambda] J_\lambda + [\kappa_\xi(\theta_\xi - r) - \pi_\xi \xi] J_\xi + J_t - \lambda V J_V - \xi J = rJ. \tag{21}$$

In PDE (21), the drift of all risk factors (V, r, λ, ξ) can now be considered risk-adjusted by the appropriate market price of risk, so that the drift of the claim (LHS of the PDE) equates to the yield of a risk-free asset, *i.e.*, the interest rate. The value of any claim depending on the four risk factors is the solution of PDE (21), given the terminal condition and the boundary conditions provided by the pay-out at expiry and, possibly, other contract terms.

Revenue-Based Zero-Coupon Bond

We will price a revenue-based zero-coupon bond for the different variations of RBF contracts examined above with respect to credit risk. We start with the Full Recourse clause, and then analyse the contracts with the Limited Recourse clause. In both cases, we will consider whether the invoice assignment determines how the repayment is made.

Full Recourse Clause

Base Case

We define the revenue-based zero-coupon bond as an asset paying a fraction ω of the revenues $V(T)$ collected during the reference period ending at time T. Let t be the evaluation time, and let $H(V, r, \lambda, t, T)$ denote the value at time t of the revenue-based zero-coupon bond: it is the solution of PDE (21) with a terminal condition equal to the pay-out. The solution can be expressed as an expectation under the risk-neutral measure Q:

$$H(V, r, \lambda, t, T) = \mathbf{E}^Q \left[e^{-\int_t^T r(s)ds} \omega V(T) \mathbf{1}_{\{\tau > T\}} \right].$$

The indicator function $\mathbf{1}_{\{\tau>T\}}$ accounts for the default of the debtor company, so that the revenue-based zero-coupon bond pays out the fraction of the revenue only if the company survives up to time T.
The expected value $H(V, r, \lambda, t, T)$ is:[1]

$$H(V, r, \lambda, t, T) = H^{FR}(t, T) = \omega P(r, t, T) V_t N(\lambda, t, T), \tag{22}$$

where N(t,T) is:

$$N(\lambda, t, T) = C(t, T) e^{-\lambda(t) D(t,T)} e^{(\mu - \pi_V)(T-t)},$$

$$C(t, T) = \left[\frac{2\phi e^{\frac{(\kappa_\lambda + \phi + \pi_\lambda)(T-t)}{2}}}{(\kappa_\lambda + \phi + \pi_\lambda)\left(e^{\phi(T-t)} - 1\right) + 2\phi} \right]^{\frac{2\kappa_\lambda \theta_\lambda}{\sigma_\lambda^2}},$$

$$D(t, T) = \frac{2\left(e^{\phi(T-t)} - 1\right)}{(\kappa_\lambda + \phi + \pi_\lambda)\left(e^{\phi(T-t)} - 1\right) + 2\phi},$$

$$\phi = \sqrt{(\kappa_\lambda + \pi_\lambda)^2 + 2\sigma_\lambda^2}.$$

The quantity $N(\lambda, t, T)$ includes the survival probability of the debtor company, so that we can also write:

$$N(\lambda, t, T) = \mathbf{SP}(t, T) e^{(\mu - \pi_V)(T-t)},$$

where $\mathbf{SP}(t, T) = C(t, T) e^{-\lambda(t) D(t,T)}$.
The quantity $P(r, t, T)$ is the price of an interest rate zero-coupon bond:

$$P(r, t, T) = A(t, T) e^{-r(t) B(t,T)},$$

$$A(t, T) = \left[\frac{2\gamma e^{\frac{(\kappa_r + \gamma + \pi_r)(T-t)}{2}}}{(\kappa_r + \gamma + \pi_r)\left(e^{\gamma(T-t)} - 1\right) + 2\gamma} \right]^{\frac{2\kappa_r \theta_r}{\sigma_r^2}},$$

$$B(t, T) = \frac{2\left(e^{\gamma(T-t)} - 1\right)}{(\kappa_r + \gamma + \pi_r)\left(e^{\gamma(T-t)} - 1\right) + 2\gamma},$$

$$\gamma = \sqrt{(\kappa + \pi_r)^2 + 2\sigma_r^2}.$$

The proof is in the Appendix "Proof of the Formulae for the Price of a Revenue-Based

[1] The solution for the expected value can be derived using standard techniques for affine processes and properties of the square-root mean-reverting SDE.

Zero-Coupon Bond".

Assignment of Invoices
We define the revenue-based zero-coupon bond as an asset paying a fraction ω of the revenues at time $V(T)$. Let t be the evaluation time: the pay-out is given by the payment of one or more invoices by the representative client company; the invoices are issued before T with expiry at time $S \geq T$ (T is the end of the reference period of the RBF contract) and are transferred at T to the buyer of the zero-coupon bond.
If the debtor company has not defaulted before T, the transfer of the invoices is possible; otherwise, the revenue-based zero-coupon bond expires worthless. Additionally, with the Full Recourse clause in force, if the client of the company (*i.e.*, the payer of the invoices) defaults, the lender may request full payment from the debtor company, so that its survival up to time S is what really matters in the evaluation of the revenue-based zero-coupon bond, without any need to account for the client's default risk.
Let us denote the value at time t of this revenue-based zero-coupon bond by $H(V, r, \lambda, t, T, S)$: it is the solution of PDE (21) with terminal condition equal to the pay-out. The solution can be expressed as an expectation under the risk-neutral measure Q:

$$H(V,r,\lambda,t,T,S) = \mathbf{E}^Q\left[e^{-\int_t^S r(s)ds}\omega V(T)\mathbf{1}_{\{\tau>S\}}\right].$$

The expected value $H(V, r, \lambda, \xi, t, T, S)$ is

$$\begin{aligned} H(V,r,\lambda,\xi,t,T,S) &= H^{FRA}(t,T,S) \\ &= \omega P(r,t,S)V_t\mathbf{SP}(t,S)e^{(\mu-\pi_V)(T-t)}, \end{aligned} \quad (23)$$

where $P(r, t, S)$ and $\mathbf{SP}(t, S)$ are the same functions provided above for the Base case. The proof in the Appendix "Proof of the Formulae for the Price of a Revenue-Based Zero-Coupon Bond".

Limited Recourse Clause

When the RBF contract provides only for Limited Recourse, we need to consider only the case when the repayment is made by assigning to the lender the invoices issued by the debtor company during the reference period. This is addressed in what follows.

Assignment of Invoices
As before, we define the revenue-based zero-coupon bond as an asset paying a fraction ω of the revenues at time $V(T)$: the pay-out is given by the payment of one or more invoices by the representative client company; the invoices are issued before T with expiry at time $S \geq T$ and they are transferred in T to the lender.
Also in this case, only if the debtor company did not default before T the transfer of the invoices is possible, but since no recourse is allowed if the client defaults, only its survival between time T, when the invoices are assigned, and time S, when they are paid. So, the debtor's default is relevant only if it happens between the evaluation time t and the assignment time T, whereas the client's default is relevant between T and S.
The solution of the PDE (21) is expressed as an expectation under the risk-neutral measure Q:

$$H(V,r,\lambda,t,T,S) = \mathbf{E}^Q\left[e^{-\int_t^S r(s)ds}\omega V(T)\mathbf{1}_{\{\tau^c>S|\tau^c>T\}}\right]. \quad (24)$$

The expected value $H(V, r, \lambda, \xi, t, T, S)$ is

$$\begin{aligned} H(V,r,\lambda,\xi,t,T,S) &= H^{LRA}(t,T,S) \\ &= \omega V_t P(r,t,S)N(\lambda,t,T)M(\xi,t,T,S). \end{aligned} \quad (25)$$

The quantity $P(r, t, S)$ and $N(\lambda, t, T)$ are the functions provided above, whereas the quantity $M(\xi, t, T, S)$ is the survival probability of the client company in the period

$[T, S]$:

$$M(\xi, t, T, S) = \mathbf{SP}^c(T, S) = E(T, S)e^{-\xi(t)\frac{\eta F(T,S)e^{-\kappa_\xi(T-t)}}{\eta+F(T,S)}}, \tag{26}$$

$$E(T, S) = \left[\frac{2\psi e^{\frac{(\kappa_\xi+\psi+\pi_\xi)(S-T)}{2}}}{(\kappa_\xi + \psi + \pi_\xi)\left(e^{\psi(S-T)} - 1\right) + 2\psi}\right]^{\frac{2\kappa_\xi\theta_\xi}{\sigma_\xi^2}} \cdot \left(\frac{\eta}{\eta + F(T, S)}\right)^{\frac{2\kappa_\xi\theta_\xi}{\sigma_\xi^2}},$$

$$F(T, S) = \frac{2\left(e^{\psi(S-T)} - 1\right)}{(\kappa_\xi + \psi + \pi_\xi)\left(e^{\psi(T-S)} - 1\right) + 2\psi},$$

$$\eta = \frac{2(\kappa_\xi + \pi_\xi)}{\sigma_\xi^2(1 - e^{-(\kappa_\xi+\pi_\xi)(S-t)})},$$

$$\psi = \sqrt{(\kappa_\xi + \pi_\xi)^2 + 2\sigma_\xi^2}.$$

When the client is a specific company and not a generic client company, $M(\xi, t, T, S)$ modifies as:

$$M(\xi, t, T, S) = \frac{\mathbf{SP}^c(t, S)}{\mathbf{SP}^c(t, T)}.$$

The proof is in the Appendix "Proof of the Formulae for the Price of a Revenue-Based Zero-Coupon Bond".

Revenue-Based Bond

A revenue-based bond is a claim depending on the future stream of revenues of the debtor company. The stream may vary depending on the terms of the contract that we analysed earlier.
The amount R_{t_0}, defined at time t_0 (which is also the evaluation time), must be reimbursed in variable and stochastic instalments equal to a fraction ω of the future revenues at times, until the amount is fully repaid. At any future date t_i, $i \in \{1, \ldots, n^*, N\}$, R_{t_i} is the outstanding balance to be repaid, with $R_{t_{n^*}} = 0$ (*i.e.*, the outstanding balance is fully repaid by the fraction of the revenues). If a term date t_N is set, then the outstanding amount is fully repaid on that date, so also in this case $R_{t_N} = 0$. Clearly, it is possible that the debt is repaid earlier than the term date, *i.e.*: $t_{n^*} \le t_N$. The repayment may or may not be made by assigning invoices to the buyer of the bond (the lender), and the contract may provide for the Full or Limited Recourse clause.
It should be noted that the last date t_{n^*} cannot be known with certainty if a term date is not set in the contract, since the full repayment of the debt depends on the future level of revenues.
Finally, we will also consider the possibility that the contract provides for a revenue floor, so that on each date t_i, repayment occurs only if the revenues of the reference period are greater than a given amount Z, *i.e.*, $V(t_i) > Z$.
As we did for the revenue-based zero-coupon bond, we will present evaluation formulae for the different RBF contract terms.

Full Recourse Clause

Base Case

The stream of cash flows of a revenue-based bond is the sum of the fraction ω of future revenues referring to the reference periods ending on dates $\{t_1, t_2, \dots, t_{n^*}, t_N\}$; each payment reduces the outstanding debt. At any time t_i, the payment is simply the outstanding debt if this is smaller than the fraction of revenues that should be paid. On the other hand, if t_i is also the term date t_N for the repayment, the cash flow is again the outstanding debt, regardless of how much the fraction of revenues equals at that time. Additionally, on each repayment date t_i, revenues must be above the floor Z. Formally, considering all this, the cash flow K_{t_i} at time t_i can be written:

$$
\begin{aligned}
K_{t_i} &= \min\left[\omega V^*(t_i), R_{t_{i-1}}\right]\mathbf{1}_{\{V(t_i)>Z\}} \\
&+ \left[R_{t_{i-1}} - \min\left[\omega V^*(t_i), R_{t_{i-1}}\right]\right]\mathbf{1}_{\{t_i=t_N\}} \\
&= \left[\omega V^*(t_i) - \max\left[\omega V^*(t_i) - R_{t_{i-1}}, 0\right]\right] \\
&\cdot \mathbf{1}_{\{V(t_i)>Z\}} \\
&+ \left[R_{t_{i-1}} - \left[\omega V^*(t_i)\right.\right. \\
&\left.\left. + \max\left[\omega V^*(t_i) - R_{t_{i-1}}, 0\right]\right]\mathbf{1}_{\{V(t_i)>Z\}}\right] \\
&\cdot \mathbf{1}_{\{t_i=t_N\}},
\end{aligned}
$$

where $R_{t_{i-1}}$ is the amount of the outstanding debt at time t_{i-1}. By setting R_{t_0} equal to the initial amount lent to the debtor company, the outstanding debt at any time t_i can be recursively calculated as:

$$
R_{t_i} = R_{t_{i-1}} - K_{t_i}.
$$

It should be noted that the quantity K_{t_i}, and hence R_{t_i}, are stochastic, since they depend on the volume of revenues generated by the business activity of the debtor. We are not considering any default risk for the moment, as we want to ascertain how long it takes to repay the debt given the revenues generated in the future. The expected payment K_{t_i} is:

$$
\begin{aligned}
\mathbf{E}^{\mathbb{Q}}[K_{t_i}] &= \mathbf{E}^{\mathbb{Q}}\left[\min\left[\omega V^*(t_i), R_{t_i}\right]\mathbf{1}_{\{V(t_i)>Z\}}\right. \\
&+ \left[R_{t_{i-1}} - \min\left[\omega V^*(t_i), R_{t_{i-1}}\right]\mathbf{1}_{\{V(t_i)>Z\}}\right] \\
&\left.\cdot \mathbf{1}_{\{t_i=t_N\}}\right] \\
&= \mathbf{E}^{\mathbb{Q}}\left[\left[\omega V^*(t_i) - \max\left[\omega V^*(t_i) - R_{t_{i-1}}, 0\right]\right]\right. \\
&\cdot \mathbf{1}_{\{V(t_i)>Z\}} + \left[R_{t_{i-1}} - \left[\omega V^*(t_i)\right.\right. \\
&\left.\left. + \max\left[\omega V^*(t_i) - R_{t_{i-1}}, 0\right]\right]\mathbf{1}_{\{V(t_i)>Z\}}\right] \\
&\left.\cdot \mathbf{1}_{\{t_i=t_N\}}\right],
\end{aligned}
\tag{27}
$$

which can be explicitly computed as:

$$
\begin{aligned}
\mathbf{E}^{\mathbb{Q}}[K_{t_i}] &= \left[\omega V(t_0)e^{(\mu-\pi_V)(t_i-t_0)}\right. \\
&\left. - \mathcal{O}(V^*(t_i), R_{t_{i-1}}, t_0, t_i)\right]\mathbf{PF}(t_0, t_i) \\
&- \left[\mathbf{E}^{\mathbb{Q}}[R_{t_{i-1}}] + \left[\omega V(t_0)e^{(\mu-\pi_V)(t_i-t_0)}\right.\right. \\
&\left.\left. - \mathcal{O}(V^*(t_i), R_{t_{i-1}}, t_0, t_i)\right]\mathbf{PF}(t_0, t_i)\right]\mathbf{1}_{\{t_i=t_N\}},
\end{aligned}
\tag{28}
$$

where

$$
\begin{aligned}
&\mathcal{O}(V^*(t_i), R_{t_{i-1}}, t_0, t_i) = \\
&\quad \mathbf{E}^{\mathbb{Q}}\left[\max\left[\omega V^*(t_i) - R_{t_{i-1}}, 0\right]\right].
\end{aligned}
\tag{29}
$$

The solution to (29) is based on the Black&Scholes pricing formula for a call option, and it is:

$$
\begin{aligned}
&\mathcal{O}(V^*(t_i), R_{t_{i-1}}, t_0, t) = \\
&= \left[\omega V^*(t_0)e^{[\mu-\pi_V](t_i-t_0)}N(d_1) - R_{t_{i-1}}N(d_2)\right],
\end{aligned}
$$

with $N(x)$ is the normal distribution function in x and:

$$
\begin{aligned}
d_1 &= \frac{\ln\frac{\omega V^*(t_0)}{R_{t_{i-1}}} + [\mu - \pi_V + \frac{1}{2}\sigma_v^2](t_i - t_0)}{\sigma_v\sqrt{t_i - t_0}}, \\
d_2 &= d_1 - \sigma_v\sqrt{t_i - t_0}.
\end{aligned}
$$

The quantity $\mathbf{PF}(t_0, t_i)$ is the (risk-neutral) probability calculated at t_0 that the revenues at the end of the reference period t_i are higher than the floor Z. The explicit formula is:

$$\mathbf{PF}(t_0,t_i) = \mathbf{E}^{Q}\Big[V(t_i) > Z\Big] = N(d_3), \quad (30)$$

with:

$$d_3 = \frac{\ln\frac{V^*(t_0)}{Z} + [\mu - \pi_V - \frac{1}{2}\sigma_v^2](t_i - t_0)}{\sigma_v\sqrt{t_i - t_0}}. \quad (31)$$

The expected outstanding amount R_{t_i} is then simply:

$$\mathbf{E}^{Q}[R_{t_i}] = \mathbf{E}^{Q}[R_{t_{i-1}} - K_{t_i}], \quad (32)$$

recursively computed. For the valuation of the bond below, the date t_{n^*}-the date when the outstanding debt becomes nil and is fully repaid-is calculated by means of equations (28) and (32). These are used for all cases of RBF contracts we are analysing. The expected present value at time t_0 of the stream of cash flows is the value of a revenue-based bond, defined as:

$$B^{FR}(R_{t_0},t_0,t_1,t_N) = \mathbf{E}^{Q}\left[\sum_{i=1}^{N} e^{-\int_{t_0}^{t_i} r(s)ds} K_{t_i}\mathbf{1}_{\{\tau>t_i\}}\right], \quad (33)$$

where t_1 is the first date when repayment starts and t_N is the term date (if no term date is set, then $t_N = \infty$, so that the summation ends at t_{n^*}, or the earliest time when the outstanding balance of the debt is fully repaid, $R_{t_{n^*}} = 0$). It is worth noting that the debt may be fully repaid before the term date t_N, so that some addends of the summation will be nil. The value $B^{FR}(R_{t_0},t_0,t_1,t_N)$ should correspond to the amount actually lent to the debtor company at time t_0, given the contract terms. This bond does not pay any coupon, but only stochastic instalments, depending on the revenues, that are used to repay R_{t_0}. The interest rate and the compensation for credit risk are implicit in the difference between the lent amount $B(R_{t_0},t_0,t_1,s)$ and the amount R_{t_0} that must be repaid. By calculating the expectations, we get the explicit formula:

$$\begin{aligned} B^{FR}(R_{t_0},t_0,t_1,t_N) &= \sum_{i=1}^{N}\Big[\big[H^{FR}(t_0,t_i) \\ &- P(r,t_0,t_i)\mathcal{O}(V^*(t_i),R_{t_{i-1}},t_0,t_i)\big]\mathbf{PF}(t_0,t_i)\mathbf{SP}(t_0,t_i)\Big] \\ &+ P(r,t_0,t_N)\Big[\mathbf{E}^{Q}[R_{t_{N-1}}] - \big[H^{FR}(t_0,t_{N-1}) \\ &- \mathcal{O}(V^*(t_N),R_{t_{N-1}},t_0,t_N)\big]\mathbf{PF}(t_0,t_i)\Big]\mathbf{SP}(t_0,t_N). \end{aligned} \quad (34)$$

Assignment of Invoices

The stream of cash flows of a revenue-based bond is the same as above: in this case, t_{n^*} is the earliest between the time when the outstanding balance of the debt is fully repaid, $R_{t_{n^*}} = 0$, and the term date t_N, when the outstanding balance must be paid in full anyway. Compared with the base case, the only difference lies in the occurrence of the cash flows, each one at a period s after the relevant end date of the reference period (for simplicity's sake, s is constant but it can be made also variable without any substantial change to the formulae below). The invoices are still related to the revenues generated in each reference period.

Equations (28) and (32) are used to calculate the expected repayments and outstanding debt at each period, and the date t_{n^*} when the debt is expected to be fully repaid. The valuation formula should be modified in two ways: first, the discount of the expected cash flows is between t_0 (the evaluation date) and the payment dates $\{t_1+s, t_2+s, \ldots, t_{n^*}+s, t_N+s\}$; second, the debtor's survival probability is also calculated up to the payment dates, since it is liable for the payment in case the client misses the payment. All the remaining terms are the same as in the base case.

The value of the revenue-based bond is the expected present value at time t_0:

$$B^{FRA}(R_{t_0}, t_0, t_1, t_N, s) = \mathbf{E}^Q \left[\sum_{i=1}^{N} e^{-\int_{t_0}^{t_i+s} r(s)ds} K_{t_i} \mathbf{1}_{\{\tau > t_i + s\}} \right], \quad (35)$$

and the explicit evaluation formula is:

$$B^{FRA}(R_{t_0}, t_0, t_1, t_N, s) = \sum_{i=1}^{n^*} \Big[[H^{FRA}(t_0, t_i, t_i + s) - P(r, t_0, t_i + s) \cdot \mathcal{O}(V^*(t_i), R_{t_{i-1}}, t_0, t_i)] \mathbf{PF}(t_0, t_i) \mathbf{SP}(t_0, t_i + s) \Big] + \Big[P(r, t_0, t_N + s) \mathbf{E}^Q[R_{t_{N-1}}] - [H^{FRA}(t_0, t_N, t_N + s) - P(r, t_0, t_N + s) \cdot \mathcal{O}(V^*(t_N), R_{t_{N-1}}, t_0, t_N)] \mathbf{PF}(t_0, t_i) \Big] \cdot \mathbf{SP}(t_0, t_N + s). \quad (36)$$

Limited Recourse Clause

As before, only the case when invoices are assigned has to be considered.

Assignment of Invoices

The stream of cash flows of a revenue-based bond is the sum of the fraction ω of future invoices issued in the reference periods and paid at the expiry dates, assumed to be a constant interval s after the ending dates $\{t_1, t_2, \dots, t_{n^*}, t_N\}$ of the reference periods. This time, we consider that the debtor company survives up to the ending dates of the reference periods $\{t_1, t_2, \dots, t_{n^*} + s, t_N\}$, and that the representative client company survives between each ending period and the payment date $[t_i, t_i + s]$, for $i \in \{1, \dots, n^*, N\}$.

Equations (28) and (32) are also used in this version of the RBF contract, whereas the value of the revenue-based bond is:

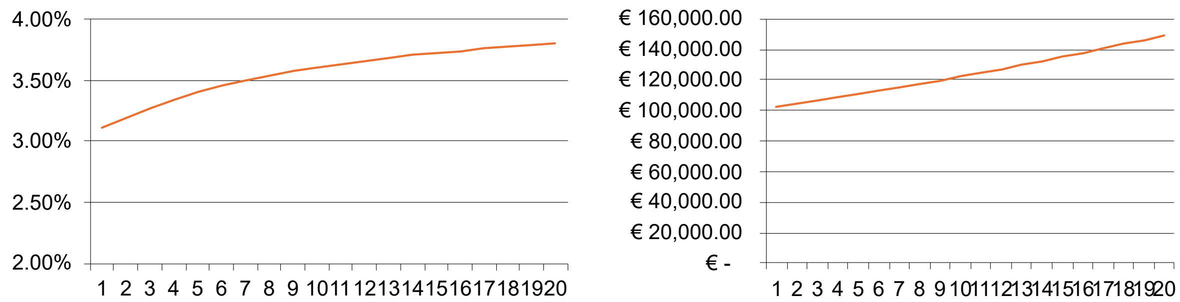
$$B^{LRA}(R_{t_0}, t_0, t_1, t_N, s) = \mathbf{E}^Q \left[\sum_{i=1}^{N} e^{-\int_{t_0}^{t_i+s} r(s)ds} K_{t_i} \mathbf{1}_{\{\tau > t_i\}} \cdot \mathbf{1}_{\{\tau^c > t_i + s | \tau^c > t_i\}} \right].$$

The explicit value is then:

$$B^{LRA}(R_{t_0}, t_0, t_1, t_N, s) = \sum_{i=1}^{n^*} \Big[[H^{LRA}(t_0, t_i, t_i + s) - P(r, t_0, t_i + s) \mathcal{O}(V^*(t_i), R_{t_{i-1}}, t_0, t_i)] \cdot \mathbf{PF}(t_0, t_i) \mathbf{SP}(t_0, t_i) \mathbf{SP}^c(t_i, t_i + s) \Big] + + P(r, t_0, t_N + s) \Big[\mathbf{E}^Q[R_{t_{N-1}}] + - [H^{LRA}(t_0, t_N, t_N + s) + - \mathcal{O}(V^*(t_N), R_{t_{N-1}}, t_0, t_N)] \mathbf{PF}(t_0, t_i) \Big] \cdot \mathbf{SP}(t_0, t_N) \mathbf{SP}^c(t_N, t_N + s).$$

Interest Rate		Debtor's Default Intensity		Client's Default Intensity		Revenues	
r_0	3.00%	λ	3.00%	ξ	4.00%	$V(t_0)$	€ 100,000.00
κ_r	0.15	κ_λ	0.2	κ_ξ	0.2	μ	5.00%
θ_r	5%	θ_λ	3.50%	θ_ξ	5.00%	π_v	3.00%
σ_r	7.50%	σ_λ	6.00%	σ_ξ	6.00%	σ_v	3.0%

FIGURE 8: Parameters of the risk factors' dynamics.



(a) Interest Rate Zero-Rate Curve. **(b)** Evolution of expected revenues

FIGURE 9: Interest rates and revenues.

Application of the Framework

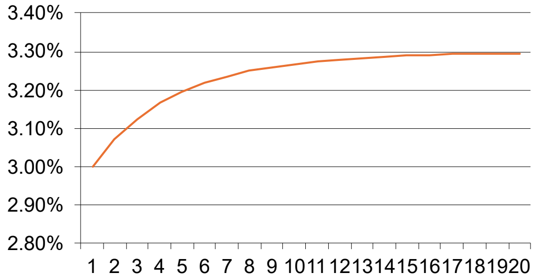
We apply the framework outlined above to a simulated environment. The parameters used for the dynamics of the risk factors introduced in Section "Modelling the Revenues of the Debtor Company and Other Relevant Risk Factors" are shown in Table 8. Figure 9 shows the resulting risk-free zero-rate term structure and the evolution of the expected revenues over a 20-year horizon; Table 18 in Appendix "Revenues, Expected Repayments and Oustanding Debts" shows also the values of the revenues generated by different values of the drift parameter μ. The term structures of the (conditional) 1-year default probability for the debtor company and the representative client company are shown in Figure 10. A final note on the market risk parameters: we set only the parameter for the revenues $\pi_V = 3.0\%$, whereas we assume that for all others risk factors they are equal to 0.

Given this set of parameters, we will now price the value of a revenue-based bond under different assumptions for the drift μ of the revenue dynamics in Equation (14). This represents the expected annual percentage variation of the revenues in the reference period. We start by analysing a revenue-based contract in the base case with full recourse: in Table 11, we show the main contract terms and the value of the revenue-based bond, the yield to maturity, the multiple of the initial lent amount (*i.e.*, the value of the bond) that must be repaid by the debtor, and the duration of the bond. The terms of the contracts are the same under all possible values of the drift μ: the lender has the right to receive a fraction $\omega = 20\%$ of the revenues generated in each future reference period, assuming current revenues $V(t_0) = 100{,}000.00$. The contract provides for no term date and no floor on the revenues below which repayment does not occur.

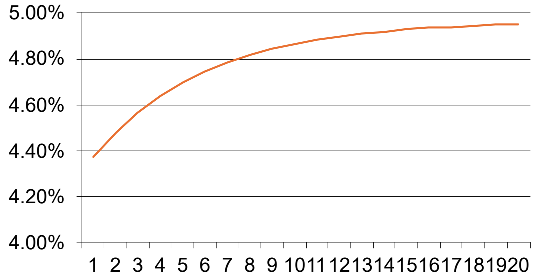
The results show that the present value of the revenue-based bond (which is the fair amount that the lender should grant to the debtor company at the start of the revenue-based contract) increases with the value of the drift μ. On the other hand, the duration is inversely related to the level of μ, whereas the yield to maturity[2] and the multiple are quite stable. In Figure 12, the expected repayments K_{t_i} and the expected outstanding debt R_{t_i} are shown for the different levels of μ. In Table 19 in Appendix , we show the values of the expected repayments and outstanding debt.

Next, we investigate the value of the revenue-based bond under different assumptions for the expected revenue growth rate μ, by adding a new term to the contract: a floor set at 80,000.00. If the revenues fall below this level, no repayment occurs. Table 13 and Figure 14 show the results as before.

[2]The yield to maturity is calculated in the usual way as the single rate that equates the (compound) discounted expected cash flows to the value of the bond.



(a) One-year default rates' curve of the debtor company.

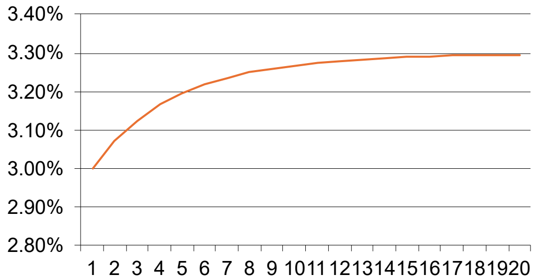


(b) One-year default rates' curve of the client company.

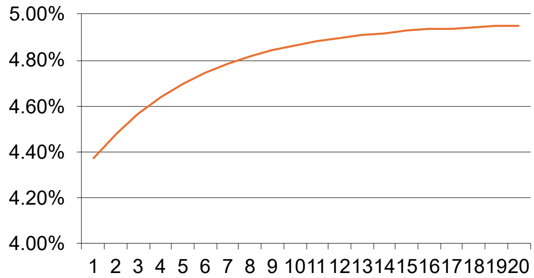
FIGURE 10: Default rates.

	μ			
	-5%	0%	5%	10%
V(t₀)	€ 100,000.00	€ 100,000.00	€ 100,000.00	€ 100,000.00
ω	20.00%	20.00%	20.00%	20.00%
Term Date	-	-	-	-
Floor	-	-	-	-
B	€ 79,853.80	€ 81,770.52	€ 83,237.13	€ 84,429.39
YTM	6.71%	6.67%	6.63%	6.60%
Multiple	1.25	1.22	1.20	1.18
Duration	3.34	3.03	2.79	2.60

FIGURE 11: Contract terms and value of a revenue-based bond with risk and yield metrics in the base case contract with no term date and with no revenues' floor.



(a) Expected repayments K_{t_i}.



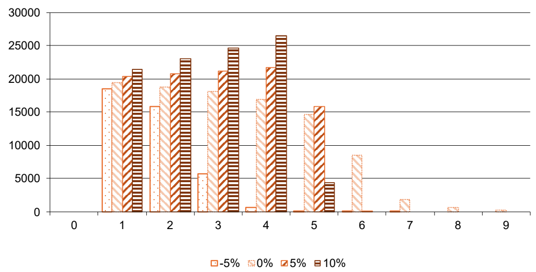
(b) Expected outstanding debt R_{t_i}.

FIGURE 12: Expected repayments and outstanding debt amounts for different levels of the drift parameter μ, in the base case contract with no term date and no revenues' floor.

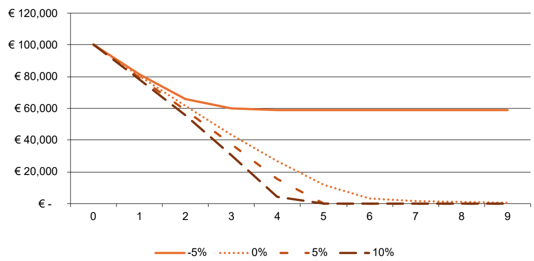
It is interesting to analyse the case when the revenue growth rate is negative, −5.0%. As shown in Figure 14, and confirmed by Table 20 in Appendix , the expected repayment stops after five years because expected revenues remain below the floor of 80,000.00. Consequently, the multiple of 2.73 on the value of the bond (36,638.30) is never fully repaid. Despite this partial recovery of the expected repaid amount (100,000.00), the lender does not suffer a negative return on the investment. In fact, it earns a total expected cash flow of 40,878.00, corresponding to an expected positive yield of 6.48%. The framework accounts for the decline in revenues and the floor, and sets the value of

	μ			
	-5%	0%	5%	10%
$V(t_0)$	€ 100,000.00	€ 100,000.00	€ 100,000.00	€ 100,000.00
ω	20.00%	20.00%	20.00%	20.00%
Term Date	-	-	-	-
Floor	€ 80,000.00	€ 80,000.00	€ 80,000.00	€ 80,000.00
B	€ 36,638.30	€ 81,065.12	€ 83,237.13	€ 84,429.39
YTM	6.48%	6.68%	6.63%	6.60%
Multiple	2.73	1.23	1.20	1.18
Duration	1.69	3.08	2.79	2.60

FIGURE 13: Contract terms and value of a revenue-based bond with risk and yield metrics in the base case contract with no term date and with a revenues' floor.



(a) Expected repayments K_{t_i}.



(b) Expected outstanding debt R_{t_i}.

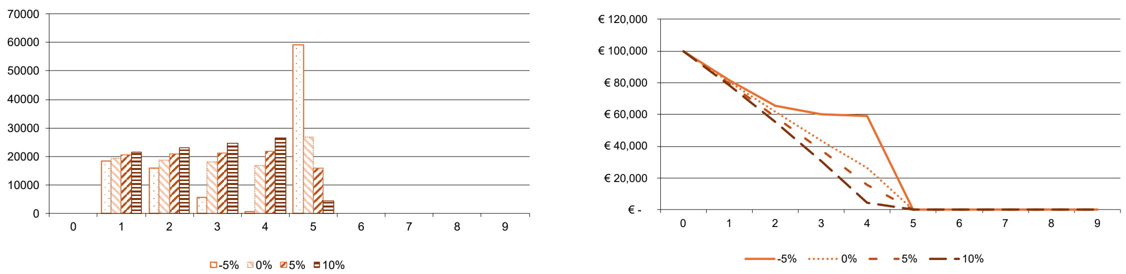
FIGURE 14: Expected repayments and outstanding debt amounts for different levels of the drift parameter μ, in the base case contract with no term date and with a revenues' floor.

the revenue-based bond at 36,638.30, with a very high multiple. These evaluation outcomes make the investment profitable for the lender on an expectation basis.
Partial recovery of the multiple amount also occurs under the assumption of a revenue growth rate of 0.0%. In this case, the floor is never breached, so repayments never stop. Nonetheless, the percentage $\omega = 20\%$ of revenues, given their expected future evolution, is not sufficient to fully repay the multiple amount. Again, this does not imply that the lender's expected return on the investment is negative; on the contrary, the framework will provide results that keep practically constant the expected yield-to-maturity under all circumstances.
Finally, we examine the case where the contract provides for a term date of five years and a floor set at 80,000.00. Table 15 and Figure 16 show the results. The floor does not affect the full repayment in this case, since at the end of year 5 the entire outstanding debt must be reimbursed by the debtor company. The duration and the yield to maturity of the investment are not substantially different from the case where no term date and no floor applied. Thus, the introduction of a term date almost completely eliminates the effect of the floor on the risk and return metrics of the investment, although the floor may still be relevant to the debtor company as it can mitigate cash outflows in the event of steeply declining revenues. For the exact values of the expected repayments and outstanding amounts, see Table 21 in Appendix "Revenues, Expected Repayments and Oustanding Debts".
The two remaining cases of contracts are left to analyse, *i.e.*, full recourse with as-

	μ			
	-5%	0%	5%	10%
$V(t_0)$	€ 100,000.00	€ 100,000.00	€ 100,000.00	€ 100,000.00
ω	20.00%	20.00%	20.00%	20.00%
Term Date	5	5	5	5
Floor	€ 80,000.00	€ 80,000.00	€ 80,000.00	€ 80,000.00
B	€ 79,293.51	€ 82,132.20	€ 83,237.13	€ 84,429.39
YTM	6.71%	6.65%	6.63%	6.60%
Multiple	1.26	1.22	1.20	1.18
Duration	3.47	2.98	2.79	2.60

FIGURE 15: *Contract terms and value of a revenue-based bond with risk and yield metrics in the base case contract with a term date and with a revenues' floor.*

signment of invoices and limited recourse with assignment of invoices. In Table 17, we show the main results for the case where $\mu = 5.0\%$, and no term date or revenue floor applies. It is easy to verify that there is only a small difference in the value of the revenue-based bond, while the other metrics are almost identical across all contract types. We will not reproduce the detailed analysis for these two cases, as, all else being equal, the results are essentially the same as in the base case.



(a) Expected repayments K_{t_i}.

(b) Expected outstanding debt R_{t_i}.

FIGURE 16: Expected repayments and outstanding debt amounts for different levels of the drift parameter μ, in the base case contract with a term date and with a revenues' floor.

	Full Recourse	Full Rec. With Invoice Assig.	Limited Rec. with Invoice Assig.
V(t_0)	€ 100,000.00	€ 100,000.00	€ 100,000.00
ω	20.00%	20.00%	20.00%
Term Date	-	-	-
Floor	-	-	-
B	€ 83,237.13	€ 81,866.65	€ 81,585.42
YTM	6.60%	6.60%	6.60%
Multiple	1.20	1.22	1.23
Duration	2.79	2.79	2.79

FIGURE 17: Contract terms and value of a revenue-based bond with risk and yield metrics for the three types of contracts.

Conclusions

We presented an evaluation framework for revenue-based contracts. We included several common contract terms observed in practice, and we also considered a less common case that we deem interesting: the assignment of invoices related to the revenues of the relevant reference period. The framework is sufficiently rich to capture the main risks in all possible variations of the contract terms, yet it remains tractable, with all formulae expressed in closed form.
The availability of a framework such as the one we presented allows for proper assessment of the financial and credit risks borne by the lender. This should also attract more investors to this alternative lending space, thereby enlarging funding opportunities for small and medium-sized enterprises.

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Annex

Proof of the Formulae for the Price of a Revenue-Based Zero-Coupon Bond

Full Recourse Clause

Base Case

The price at time t of a revenue-based zero-coupon bond $H(t,T)$ expiring in T, is the solution of the PDE (21) with terminal condition $H(V,r,\lambda,T,T) = \omega V(T)$. It is well known that the solution can be represented as an expectation[3] (under the risk-neutral measure Q):

$$H(V,r,\lambda,t,T) = \mathbf{E}^{Q}\left[e^{-\int_t^T r(s)ds}\omega V(T)\mathbf{1}_{\{\tau>T\}}\right]. \tag{37}$$

For an explicit solution, let us try with a function of the type:
$H(V,r,\lambda,t,T) = VP(r,t,T)N(\lambda,t,T)\omega$,
with terminal condition $P(r,T,T) = 1$, $N(\lambda,T,T) = 1$. By replacing it in the PDE (21) and simplifying the notation, we get:

$$\begin{aligned} &\frac{1}{2}\sigma_r^2 rP_{rr}VN\omega + \frac{1}{2}\sigma_\lambda^2\lambda N_{\lambda\lambda}VP\omega + (\mu-\pi_V)VNP\omega + [\kappa_r(\theta_r - r) - \pi_r rP_r VP\omega + \\ &+ [\kappa_\lambda(\theta_\lambda - \lambda) - \pi_\lambda\lambda]N_\lambda VP\omega + P_t VN\omega + N_t VP\omega - \lambda VPN\omega - rVPN\omega = 0. \end{aligned} \tag{38}$$

We can split the PDE (46) in three PDE's, each one equation to 0. The first one collects all the terms where the risk factor λ is involved; after dividing by $VP\omega$ we get:

$$\frac{1}{2}\sigma_\lambda^2\lambda N_{\lambda\lambda} + [\kappa_\lambda(\theta_\lambda - \lambda) - \pi_\lambda\lambda]N_\lambda + N_t - \lambda N + (\mu - \pi_V)N = 0.$$

The solution is derived by imposing the terminal condition $N(\lambda,T,T) = 1$, so that as an expectation it is:

$$N(\lambda,t,T) = \mathbf{E}^{Q}\left[\mathbf{1}_{\tau>T}e^{-\int_t^T(\mu-\pi_V)ds}\right] = \mathbf{E}^{Q}\left[e^{-\int_t^T -\lambda(s)ds}\right]e^{(\mu-\pi_V)(T-t)}.$$

It is straightforward to note that the expectation is the price of a interest rate zero-coupon bond where the discounting is given by the process $\lambda(s)$. By exploiting the result in Cox, Ingersoll and Ross [3], and by considering that if λ follows the mean reverting square-root process (17), then the solution is:

$$N(\lambda,t,T) = C(t,T)e^{-\lambda(t)D(t,T)}e^{(\mu-\pi_V)(T-t)},$$

with $C(t,T)$ and $D(t,T)$ provided in the main text. We also stress that $N(t,T)$ takes into account the survival probability of the debtor company up to time T with the terms $\mathbf{SP}(t,T) = C(t,T)e^{-\lambda(t)D(t,T)}$; the second exponential $e^{(\mu-\pi_V)(T-t)}$ considers the remaining part of the drift of the revenues.
The second PDE collects all the terms in the PDE (46) that involve the risk factor r; dividing by $VN\omega$ we get:

$$\frac{1}{2}\sigma_r^2 rP_{rr} + [\kappa_r(\theta_r - r) - \pi_r r]P_r + P_t - rP = 0, \tag{39}$$

[3] See for example Friedman [5], Theorem 5.2.

with terminal condition $P(r,S,S) = 1$, whose solution as en expectation is:

$$P(r,t,T) = \mathbf{E}^{Q}\left[e^{-\int_t^T r(s)ds}\right].$$

The explicit solution is in the main text and it is the price of a zero-coupon bond provided in Cox, Ingersoll and Ross [3]. The explicit solution is in the main text.

Assignment of Invoices

The price at time t of a revenue-based zero-coupon bond $H(V,r,\lambda,t,T,S)$ expiring in S, when invoices issued in the reference period ending in T are paid, is the solution of the PDE (21) with terminal condition $H(V,r,\lambda,S,T,S) = \omega V(T)$. It is well known that the solution can be represented as an expectation[4])under the risk-neutral measure Q):

$$H(V,r,\lambda,t,T,S) = \mathbf{E}^{Q}\left[e^{-\int_t^S r(s)ds}\omega V(T)\mathbf{1}_{\{\tau>S\}}\right]. \quad (40)$$

The explicit solution is very similar to the *Base case*, with the only difference given by the expiry of the discount factor and the survival of the debtor company up to the expiry S. The solution is

$$H(V,r,\lambda,t,T,S) = VP(r,t,S)\mathbf{SP}(t,S)e^{(\mu-\pi_V)(T-t)}\omega. \quad (41)$$

The proof follows the same steps as above, with the terminal conditions $P(r,S,S) = 1$, $N(\lambda,S,S) = 1$. In the main text the explicit formula is provided.

Limited Recourse Clause

Assignment of Invoices

The price at time t of a revenue-based zero-coupon bond $H(V,r,\lambda,T,T,S)$ expiring in S, when invoices issued in the reference period ending in T are paid, is the solution of the PDE (21) with terminal condition $H(V,r,\lambda,S,S,S) = \omega V(S)$. It is well known that the solution can be represented as an expectation[5])under the risk-neutral measure Q):

$$H(V,r,\lambda,t,T,S) = \mathbf{E}^{Q}\left[e^{-\int_t^S r(s)ds}\omega V(T)\mathbf{1}_{\{\tau^c>S|\tau^c>T\}}\right]. \quad (42)$$

As a solution, let us try again with a function of the type: $H(V,r,\lambda,t,T,S) = VP(r,t,S)N(\lambda,t,T)$ $M(\xi,t,T,S)\omega$, with terminal conditions $P(r,S,S) = 1$, $N(\lambda,T,T) = 1$ and $M(\xi,t,S,S)$. By replacing it in the PDE (21) and simplifying the notation, we get:

$$\frac{1}{2}\sigma_r^2 rP_{rr}VNM\omega + \frac{1}{2}\sigma_\lambda^2\lambda N_{\lambda\lambda}VPM\omega + \frac{1}{2}\sigma_\xi^2\xi M_{\xi\xi}VPN + (\mu-\pi_V)VNPM\omega + [\kappa_r(\theta_r - r) +$$
$$-\pi_r rP_r VPM\omega + [\kappa_\lambda(\theta_\lambda - \lambda) - \pi_\lambda\lambda]N_\lambda VPM\omega + [\kappa_\xi(\theta_\xi - r) - \pi_\xi\xi]M_\xi VPN\omega$$
$$+ P_t VNM\omega + N_t VPM\omega + M_t VPN\omega - \lambda VPNM\omega - \xi VPNM\omega - rVPNM\omega = 0. \quad (43)$$

We can split the PDE (46) in three PDE's, each one equation to 0. The first one collects all

[4]See for example Friedman [5], Theorem 5.2.
[5]See for example Friedman [5], Theorem 5.2.

the terms where the risk factor λ is involved; after dividing by $VPM\omega$ we get:

$$\frac{1}{2}\sigma_\lambda^2\lambda N_{\lambda\lambda} + [\kappa_\lambda(\theta_\lambda - \lambda) - \pi_\lambda\lambda]N_\lambda + N_t - (1+\alpha)\lambda N + (\mu - \pi_V)N = 0.$$

The solution is derived by imposing the terminal condition $N(\lambda, T, T) = 1$, and it is the same as seen before in the *Base case*, with the explicit formula in the main text.
The second PDE collects all the terms in the PDE (46) that involve the risk factor r; dividing by $VMN\omega$ we get:

$$\frac{1}{2}\sigma_r^2 r P_{rr} + [\kappa_r(\theta_r - r) - \pi_r r]P_r + P_t - rP = 0, \tag{44}$$

with terminal condition $P(r, S, S) = 1$. Once again, the solution is tha same as above and the explicit formula provided in the main text.
The third PDE collects all the terms in the PDE (46) that involve the risk factor ξ, which after dividing by $VPN\omega$ we get:

$$\frac{1}{2}\sigma_\xi^2\xi M_{\xi\xi} + [\kappa_\xi(\theta_\xi - r) - \pi_\xi]M_\xi + M_t - \xi M = 0, \tag{45}$$

with terminal condition $M(\xi, S, S, S) = 1$. The solution expressed as an expectation is:

$$M(\xi, t, T, S) = \mathbf{E}^Q\left[\mathbf{1}_{\{\tau^c > S | \tau^c > T\}}\right] = \mathbf{E}^Q\left[e^{-\int_T^S \xi(s)ds}\right].$$

It should be noted that $M(\xi, t, T, S)$ is the survival probability of the generic client company that pays the invoice. As such, we do not refer to a specific company and we can be sure that there is always a surviving generic company whose invoices are transferred to the lender. That means that the default process is in practice memoryless and we do not need to condition on the survival of the company up to time T.
The solution is the price of a zero-coupon futures price, derived by Cox, Ingersoll and Ross [4] and explicitly provided in the main text.
In some types of contracts we are not considering a generic client but a specific company, so that the default process cannot memoryless: in this case need to calculate the survival probability up to time S conditioned to he survival up to time T, s that:

$$M(\xi, t, T, S) = \frac{\mathbf{SP}^c(t, S)}{\mathbf{SP}^c(t, T)}.$$

The solution for $\mathbf{SP}^c(t, S)$ is the same derived above for the survival probability of the debtor company $\mathbf{SP}(t, T)$, where parameters of the client's default intensity ξ replace those of the debtor's default intensity λ.
The price at time t of a revenue-based zero-coupon bond $H(V, r, \lambda, T, T, S)$ expiring in S, when invoices issued in the reference period ending in T are paid, is the solution of the PDE (21) with terminal condition $H(V, r, \lambda, S, S, S) = \omega V(S)$. It is well known that the solution can be represented as an expectation[6])under the risk-neutral measure Q):

$$H(V, r, \lambda, t, T, S) = \mathbf{E}^Q\left[e^{-\int_t^S r(s)ds}\omega V(T)\mathbf{1}_{\{\tau > S\}}\right].$$

The explicit solution is very similar to the *Base case*, with the only difference given by

[6]See for example Friedman [5], Theorem 5.2.

the expiry of the discount factor and the survival of the debtor company up to th expiry S. For an explicit solution, let us try with a function of the type: $H(t,T) = VP(r,t,T)N(\lambda,t,T)M(\xi,t,T,S)\omega$, with terminal condition $P(r,S,S) = 1$, $N(\lambda,T,T) = 1$ and $M(\xi,S,S,S) = 1$. By replacing it in the PDE (21) and simplifying the notation, we get:

$$\begin{aligned}&\frac{1}{2}\sigma_r^2 rP_{rr}VNM\omega + \frac{1}{2}\sigma_\lambda^2\lambda N_{\lambda\lambda}VPM\omega + \frac{1}{2}\sigma_\xi^2\xi M_{\xi\xi}VPN + (\mu-\pi_V)VNPM\omega +\\ &+ [\kappa_r(\theta_r - r) - \pi_r rP_r VPM\omega + [\kappa_\lambda(\theta_\lambda-\lambda) - \pi_\lambda\lambda]N_\lambda VPM\omega + [\kappa_\xi(\theta_\xi - r) - \pi_\xi\xi]M_\xi VPN\omega\\ &+ P_tVNM\omega + N_tVPM\omega + M_tVPN\omega - \lambda VPNM\omega - \xi VPNM\omega - rVPNM\omega = 0.\end{aligned} \tag{46}$$

We can split the PDE (46) in three PDE's, each one equation to 0. The first one collects all the terms where the risk factor λ is involved; after dividing by $VPM\omega$ we get:

$$\frac{1}{2}\sigma_\lambda^2\lambda N_{\lambda\lambda} + [\kappa_\lambda(\theta_\lambda-\lambda) - \pi_\lambda\lambda]N_\lambda + N_t - (1+\alpha)\lambda N + (\mu-\pi_V)N = 0.$$

The solution is derived by imposing the terminal condition $N(\lambda,T,T) = 1$, so that as an expectation it is:

$$N(\lambda,t,T) = \mathbf{E}^Q\left[\mathbf{1}_{\tau>T}e^{-\int_t^T(\mu-\pi_V)ds}\right] = \mathbf{E}^Q\left[e^{-\int_t^T -\lambda(s)ds}\right]e^{(\mu-\pi_V)(T-t)}.$$

It is straightforward to note that the expectation is the price of a interest rate zero-coupon bond where the discounting is given by the process $\lambda(s)$. By exploiting the result in Cox, Ingersoll and Ross [3], and by considering that if λ follows the mean reverting square-root process (17), then the solution is:

$$N(\lambda,t,T) = C(t,T)e^{-\lambda(t)D(t,T)}e^{(\mu-\pi_V)(T-t)},$$

with $C(t,T)$ and $D(t,T)$ provided in the main text. We also stress that $N(t,T)$ takes into account the survival probability of the debtor company up to time T with the terms $\mathbf{SP}(t,T) = C(t,T)e^{-\lambda(t)D(t,T)}$; the second exponential $e^{(\mu-\pi_V)(T-t)}$ considers the remaining part of the drift of the revenues.

The second PDE collects all the terms in the PDE (46) that involve the risk factor r; dividing by $VMN\omega$ we get:

$$\frac{1}{2}\sigma_r^2 rP_{rr} + [\kappa_r(\theta_r - r) - \pi_r r]P_r + P_t - rP = 0, \tag{47}$$

with terminal condition $P(r,S,S) = 1$, whose solution as en expectation is:

$$P(r,t,S) = \mathbf{E}^Q\left[e^{-\int_t^S r(s)ds}\right].$$

The explicit solution is in the main text and it is the price of a zero-coupon bond provided in Cox, Ingersoll and Ross [3].

The third PDE collects all the terms in the PDE (46) that involve the risk factor ξ, which after dividing by $VPN\omega$ we get:

$$\frac{1}{2}\sigma_\xi^2\xi M_{\xi\xi} + [\kappa_\xi(\theta_\xi - r) - \pi_\xi]M_\xi + M_t - \xi M = 0, \tag{48}$$

with terminal condition $M(\xi, S, S, S) = 1$. The solution expressed as an expectation is:

$$M(\xi, t, T, S) = \mathbf{E}^{Q}\left[\mathbf{1}_{\{\tau^c > S | \tau^c > T\}}\right] = \mathbf{E}^{Q}\left[e^{-\int_T^S \xi(s)ds}\right].$$

It should be noted that $M(\xi, t, T, S)$ is the survival probability of the generic client company that pays the invoice. As such, we do not refer to a specific company and we can be sure that there is always a surviving generic company whose invoices are transferred to the lender. That means that the default process is in practice memoryless and we do not need to condition on the survival of the company up to time T.

The solution is the price of a zero-coupon futures price, derived by Cox, Ingersoll and Ross [4] and explicitly provided in the main text.

In some types of contracts we are not considering a generic client but a specific company, so that the default process cannot memoryless: in this case need to calculate the survival probability up to time S conditioned to the survival up to time T, *i.e.*:

$$\mathbf{E}^{Q}\left[\mathbf{1}_{\{\tau^c > S | \tau^c > T\}}\right] = \mathbf{P}(\mathbf{1}_{\{\tau^c > S | \tau^c > T\}}) = \frac{\mathbf{P}(\mathbf{1}_{\{\tau^c > S\}})}{\mathbf{P}(\mathbf{1}_{\{\tau^c > T\}})} = \frac{\mathbf{E}^{Q}\left[\mathbf{1}_{\{\tau^c > S\}}\right]}{\mathbf{E}^{Q}\left[\mathbf{1}_{\{\tau^c > T\}}\right]},$$

so that:

$$M(\xi, t, T, S) = \frac{\mathbf{SP}^c(t, S)}{\mathbf{SP}^c(t, T)}.$$

The solution for $\mathbf{SP}^c(t, S)$ is the same derived above for the survival probability of the debtor company $\mathbf{SP}(t, T)$, where parameters of the client's default intensity ξ replace those of the debtor's default intensity λ.

Revenues, Expected Repayments and Oustanding Debts

	μ			
Time	**-5%**	**0%**	**5%**	**10%**
0	€100,000	€100,000	€100,000	€100,000
1	€92,312	€97,045	€102,020	€107,251
2	€85,214	€94,176	€104,081	€115,027
3	€78,663	€91,393	€106,184	€123,368
4	€72,615	€88,692	€108,329	€132,313
5	€67,032	€86,071	€110,517	€141,907
6	€61,878	€83,527	€112,750	€152,196
7	€57,121	€81,058	€115,027	€163,232
8	€52,729	€78,663	€117,351	€175,067
9	€48,675	€76,338	€119,722	€187,761
10	€44,933	€74,082	€122,140	€201,375
11	€41,478	€71,892	€124,608	€215,977
12	€38,289	€69,768	€127,125	€231,637
13	€35,345	€67,706	€129,693	€248,432
14	€32,628	€65,705	€132,313	€266,446
15	€30,119	€63,763	€134,986	€285,765
16	€27,804	€61,878	€137,713	€306,485
17	€25,666	€60,050	€140,495	€328,708
18	€23,693	€58,275	€143,333	€352,542
19	€21,871	€56,553	€146,228	€378,104
20	€20,190	€54,881	€149,182	€405,520

FIGURE 18: Projection of expected revenues.

	μ							
	-5%		0%		5%		10%	
Time	K	R	K	R	K	R	K	R
0		€100,000		€100,000		€100,000		€100,000
1	€18,462	€81,538	€19,409	€80,591	€20,404	€79,596	€21,450	€78,550
2	€17,043	€64,495	€18,835	€61,756	€20,816	€58,780	€23,005	€55,544
3	€15,733	€48,762	€18,279	€43,477	€21,237	€37,543	€24,674	€30,871
4	€14,523	€34,239	€17,738	€25,739	€21,666	€15,877	€26,460	€4,411
5	€13,406	€20,833	€17,214	€8,525	€15,877	€0	€4,411	€-
6	€12,376	€8,457	€8,525	€0	€-	€0	€-	€-
7	€8,457	€0	€-	€0	€-	€0	€-	€-
8	€0	€0	€-	€0	€-	€0	€-	€-
9	€-	€0	€-	€0	€-	€0	€-	€-

FIGURE 19: *Expected repayments and outstanding debt amounts for different levels of the drift parameter μ, in the base case contract with no term date and no revenues' floor.*

	μ							
	-5%		0%		5%		10%	
Time	K	R	K	R	K	R	K	R
0		€100,000		€100,000		€100,000		€100,000
1	€18,462	€81,538	€19,409	€80,591	€20,404	€79,596	€21,450	€78,550
2	€15,830	€65,708	€18,834	€61,757	€20,816	€58,780	€23,005	€55,544
3	€5,711	€59,997	€18,176	€43,581	€21,237	€37,543	€24,674	€30,871
4	€727	€59,270	€16,929	€26,651	€21,666	€15,877	€26,460	€4,411
5	€51	€59,219	€14,713	€11,938	€15,877	€0	€4,411	€-
6	€3	€59,216	€8,464	€3,474	€0	€0	€-	€-
7	€0	€59,216	€1,911	€1,563	€-	€0	€-	€-
8	€0	€59,216	€633	€930	€-	€0	€-	€-
9	€0	€59,216	€266	€665	€-	€0	€-	€-
10	€0	€59,216	€130	€535	€-	€0	€-	€-
11	€0	€59,216	€70	€465	€-	€0	€-	€-
12	€0	€59,216	€40	€425	€-	€0	€-	€-
13	€0	€59,216	€23	€402	€-	€0	€-	€-
14	€0	€59,216	€14	€387	€-	€0	€-	€-
15	€-	€59,216	€9	€379	€-	€0	€-	€-
16	€-	€59,216	€5	€374	€-	€0	€-	€-
17	€-	€59,216	€3	€370	€-	€0	€-	€-
18	€-	€59,216	€2	€368	€-	€0	€-	€-
19	€-	€59,216	€1	€367	€-	€0	€-	€-
20	€-	€59,216	€1	€366	€-	€0	€-	€-

FIGURE 20: Expected repayments and outstanding debt amounts for different levels of the drift parameter μ, in the base case contract with no term date and with a revenues' floor.

μ								
	-5%		0%		5%		10%	
Time	**K**	**R**	**K**	**R**	**K**	**R**	**K**	**R**
0		€100,000		€100,000		€100,000		€100,000
1	€18,462	€81,538	€19,409	€80,591	€20,404	€79,596	€21,450	€78,550
2	€15,830	€65,708	€18,834	€61,757	€20,816	€58,780	€23,005	€55,544
3	€5,711	€59,997	€18,176	€43,581	€21,237	€37,543	€24,674	€30,871
4	€727	€59,270	€16,929	€26,651	€21,666	€15,877	€26,460	€4,411
5	€59,270	€-	€26,651	€-	€15,877	€-	€4,411	€-
6	€-	€-	€-	€-	€-	€-	€-	€-
7	€-	€-	€-	€-	€-	€-	€-	€-
8	€-	€-	€-	€-	€-	€-	€-	€-
9	€-	€-	€-	€-	€-	€-	€-	€-
10	€-	€-	€-	€-	€-	€-	€-	€-

FIGURE 21: *Expected repayments and outstanding debt amounts for different levels of the drift parameter μ, in the base case contract with a term date and with a revenues' floor.*

Multilevel Monte Carlo for Solvency II SCR: Efficient Nested Simulation

About the Authors

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The paper proposes the use of Multilevel Monte Carlo as an efficiency-enhancing method for the calculation of the Solvency Capital Requirement within the framework of internal insurance models. The computation of the Solvency Capital Requirement relies on nested Monte Carlo simulations, which become computationally prohibitive for complex portfolios. Commonly adopted proxy methods reduce execution times but introduce model risk and require a complex governance framework. The Multilevel Monte Carlo approach acts as an accelerator that does not rely on proxy functions. Instead, it employs a hierarchy of simulation levels, ranging from coarse to increasingly refined representations, in order to reduce variance and computational cost while preserving the nested simulation structure. The method is risk preserving because it avoids structural bias, ensures near optimal convergence rates, and facilitates regulatory validation through its statistical transparency. The paper also introduces an adaptive version of the Multilevel Monte Carlo method. This extension concentrates computational effort on the most critical scenarios in the tail of the loss distribution, thereby further improving efficiency in the calculation of the capital requirement.

This paper addresses the problem of calculating the Solvency Capital Requirement (SCR) under Solvency II, which requires insurance companies to hold sufficient capital to withstand extreme losses. The two-layer simulation framework underlying the capital requirement calculation consists of "real-world" simulations of risk factors and, for each of them, corresponding "risk-neutral" valuations of the portfolio. Nested simulation structures of this type are not unique to solvency applications and arise in several other areas of quantitative finance, particularly in the computation of counterparty credit risk (CCR) exposure metrics such as Expected Positive Exposure (EPE), as well as in the valuation and sensitivity analysis of XVA adjustments. While this structure ensures theoretical consistency, it entails a substantial computational burden. The paper discusses the widespread industry adoption of the Least Squares Monte Carlo (LSMC) approach, which relies on proxy functions. However, such approximations may introduce model risk, particularly in the tails of the distribution that are crucial for SCR estimation, and require specific calibration procedures and underlying structural assumptions. The core contribution of the paper is the application of the Multilevel Monte Carlo (MLMC) method, which builds a hierarchy of accuracy levels by combining many low-cost, coarse simulations with a few high-cost, precise ones, and proposing an adaptive version that concentrates computational effort on scenarios close to the critical loss threshold. The methodology is subsequently applied to a index-linked life insurance product with a minimum capital guarantee, combining financial and biometric risks within a nonlinear payoff structure. The paper concludes that MLMC preserves the transparency and accuracy of the standard Nested Monte Carlo approach while significantly reducing computational time, thereby avoiding the need for complex proxy functions. Finally, it points to the potential extension toward Multi-index Monte Carlo (MIMC) to simultaneously address multiple sources of numerical error.

SCR Computation under Solvency II

Solvency II introduced a risk-based capital regime for insurers, defining the Solvency Capital Requirement (SCR) as the capital needed to withstand a 1-in-200 year adverse event [6]. The SCR corresponds to the 99.5% one-year Value-at-Risk (VaR) of the loss in Basic Own Funds (BOF). In regulatory terms, the SCR represents the capital needed to ensure that an insurer can absorb extreme but plausible losses over a one-year horizon. Insurers may compute the SCR either via the prescribed Standard Formula or through a company-specific Internal Model, subject to supervisory approval. Within an approved Internal Model, this requirement is computed by simulating the full distribution of one-year changes in BOF and extracting its extreme quantile under market-consistent valuation principles.

Operationally, this entails a two-layer structure. First, real-world ($\mathbb{P}$) simulations generate one-year scenarios for all material risk drivers. Second, for each scenario, the balance sheet must be revalued at the one-year horizon. While assets are often directly observable or analytically tractable, liabilities, especially those embedding financial guarantees, profit-sharing mechanisms, policyholder behaviour and management actions, rarely admit closed-form expressions. Their value at the one-year horizon is typically expressed as a conditional expectation under the risk-neutral ($\mathbb{Q}$) measure. This naturally leads to a Nested Monte Carlo (NMC) architecture, with an outer $\mathbb{P}$-simulation and an inner $\mathbb{Q}$-valuation performed for each real-world scenario.

The resulting computational burden scales proportionally to $N_{\text{out}} \times N_{\text{in}}$. Because the SCR targets a deep tail quantile, both layers must be sufficiently large to control statistical error and avoid distortions in the ordering of extreme scenarios. For realistic life portfolios with path-dependent features, brute-force nested simulation rapidly becomes impractical in production environments subject to tight reporting deadlines.

To address this issue, the industry has widely adopted proxy techniques, most notably Least Squares Monte Carlo (LSMC) following Longstaff and Schwartz [12], as well as replicating portfolio approaches. These methods replace repeated inner valuations with an approximating function calibrated on a limited training set of fully nested runs. While computationally efficient, proxy approaches introduce structural approximation risk, particularly in the tail region driving the SCR. As documented in the nested simulation literature [10, 3], tail estimation is highly sensitive to model misspecification and extrapolation. Consequently, the computational problem is partly transformed into a governance challenge, requiring extensive validation, stability analysis and documentation of approximation quality.

This paper proposes an alternative that preserves the fully simulation-based structure while materially reducing computational cost: Multilevel Monte Carlo (MLMC), pioneered by Mike Giles [7] and extended to nested risk estimation by Giles and Haji-Ali [8]. Rather than replacing the inner valuation with a parametric surrogate, MLMC applies a hierarchy of simulation accuracies to the $\mathbb{Q}$-valuation layer. Using a telescopic decomposition, most samples are allocated to inexpensive coarse approximations, while only a limited number are used to correct bias at finer levels. Under suitable conditions, this approach achieves near-optimal Monte Carlo complexity for nested problems, including tail probabilities and quantiles.

In contrast to proxy methods, MLMC does not rely on an assumed functional form for the liability value and remains structurally close to full nested simulation. This feature is particularly attractive in an Internal Model context, where transparency, statistical error control and methodological consistency with regulatory expectations are essential. Practitioner evidence in solvency applications indicates that MLMC can deliver substantial runtime reductions while maintaining valuation fidelity [2].

SCR and the Nested Monte Carlo Challenge

The Structure of SCR in an Internal Model

Let $t = 0$ denote the valuation date and $t = 1$ the one-year horizon. Define Basic Own Funds:

$$BOF_t := A_t - L_t, \tag{49}$$

where A_t is the market-consistent value of assets and L_t the market-consistent value of liabilities at time t. Define the one-year loss random variable (sign convention consistent with capital requirement):

$$\Delta BOF := BOF_0 - BOF_1. \quad (50)$$

The Internal Model SCR is:

$$SCR := VaR_{0.995}(\Delta BOF). \quad (51)$$

The definition in (51) embeds several layers of modelling complexity. First, BOF_1 is itself a random variable driven by the joint evolution of all material risk factors over the one-year horizon. These typically include market variables (interest rates, equity indices, credit spreads, implied volatilities), biometric factors (mortality, longevity, lapse), and potentially management actions and dynamic policyholder behaviour. The SCR therefore depends on the full joint distribution of these drivers under the real-world measure $\mathbb{P}$, including their nonlinear interactions. A standard simulation architecture is:

- **Outer simulation ($\mathbb{P}$).** Generate N_{out} independent scenarios of the one-year risk drivers, producing time-one states $X_1^{(i)}$. The state vector X_1 typically contains the entire term structure of interest rates, equity levels, spread curves, volatility surfaces, and relevant insurance risk factors. The outer simulation must be sufficiently rich to capture tail co-movements, as the 99.5% quantile is sensitive to joint stress configurations rather than marginal extremes alone.

- **Inner valuation ($\mathbb{Q}$).** For each $X_1^{(i)}$, compute $L_1^{(i)}$ as a market-consistent value of future cashflows conditional on the time-1 filtration:

$$L_1^{(i)} = \mathbb{E}^{\mathbb{Q}}\left[\sum_{k:t_k>1} D(1,t_k)\,\text{CF}_{t_k} \,\middle|\, X_1^{(i)} \right],$$

  where $D(1,t_k)$ is the stochastic discount factor from 1 to t_k under $\mathbb{Q}$, and CF_{t_k} are liability cashflows.
  The liability value at time 1 can be expressed as a conditional expectation under the risk-neutral measure. In general, no closed-form expression is available when cashflows depend pathwise on future economic scenarios. As a consequence, the conditional expectation must be approximated numerically, most commonly by Monte Carlo simulation under $\mathbb{Q}$. This creates a nested structure: for each outer scenario $X_1^{(i)}$, an inner simulation with N_{in} paths is required to estimate $L_1^{(i)}$.
  It is worth noting that assets may also require revaluation at $t = 1$, especially for path-dependent instruments. However, in many internal model implementations, the dominant computational burden arises from the liability side, where long-dated optionalities and management rules induce high-dimensional path dependence.

- **Quantile estimation.** Compute $\Delta BOF^{(i)}$ via (50) for all outer scenarios, then estimate $VaR_{0.995}$ from the empirical distribution. Since the target is an extreme percentile, the accuracy of the estimator depends critically on the number of outer scenarios and on the noise introduced by the inner estimations of $L_1^{(i)}$. Inner simulation error propagates nonlinearly to the quantile estimator and may induce bias if not properly controlled.

Why Nested Monte Carlo Arises

Liability valuation at time 1 is a conditional expectation under $\mathbb{Q}$. Given the state $X_1^{(i)}$, the liability value is a functional of the entire future path of risk factors beyond the one-year horizon. Whenever cashflows depend nonlinearly on future asset returns, interest rate paths, policyholder behaviour, or management rules, no tractable closed-form expression is available for this conditional expectation. As a consequence, numerical integration is required, and in practice this is almost always performed via Monte Carlo simulation under the risk-neutral measure. For complex liabilities, one therefore resorts to an inner Monte Carlo estimator (see e.g. Glasserman [9] for a comprehensive treatment of Monte Carlo methods in financial engineering):

$$\widehat{L}_1^{(i)} := \frac{1}{N_{\text{in}}} \sum_{k=1}^{N_{\text{in}}} Y^{(i,k)}, \tag{52}$$

where $Y^{(i,k)}$ is the discounted present value of future cashflows along inner path N_{in}, simulated under $\mathbb{Q}$ from the state $X_1^{(i)}$. The estimator in (52) is unbiased for the conditional expectation, with variance decreasing at rate $1/N_{\text{in}}$. However, achieving a sufficiently small conditional standard error may require a large N_{in}, especially for long-dated or highly path-dependent contracts. Since this inner valuation must be repeated for each outer scenario $i = 1, \dots, N_{\text{out}}$, the resulting structure is inherently nested (Figure 22). The overall computational cost scales as:

$$\text{Cost}_{\text{NMC}} \propto N_{\text{out}} \times N_{\text{in}},$$

up to model-dependent constants capturing projection engine complexity (e.g. number of policy model points, granularity of asset models, time discretization steps). In realistic life portfolios, both N_{out} and N_{in} must be large to control sampling error at the portfolio level, leading to substantial runtime.

The statistical challenge is further compounded by the quantile objective. The SCR targets the 99.5% tail of the loss distribution, and this tail is sensitive not only to outer sampling variability but also to noise in $\widehat{L}_1^{(i)}$. Inner simulation error may induce mis-ordering of scenarios near the tail, thereby distorting the empirical quantile and potentially introducing bias. Importantly, reducing the variance of the inner estimator uniformly across all scenarios is not necessarily optimal, since only a small fraction of outer paths ultimately determine the tail estimate.

The literature on nested risk estimation shows that naive nested Monte Carlo estimators can exhibit slow convergence for risk measures such as VaR, especially when inner noise is not properly controlled relative to the outer sample size. In particular, inefficient allocation between N_{out} and N_{in} can lead to suboptimal complexity and persistent bias effects [10, 3]. These findings motivate more refined allocation strategies and advanced variance-reduction techniques tailored to the nested structure of the SCR problem.

Least Squares Monte Carlo (LSMC) as an Industrial Compromise

LSMC originates from the Longstaff–Schwartz methodology for American options [12]. In solvency applications, the key idea is to avoid performing a full inner Monte Carlo valuation for every outer scenario. Instead, one constructs a proxy function that approximates the time-1 liability value as a function of the risk factors:

$$L_1 \approx \phi(X_1; \theta),$$

where X_1 is a vector of risk factors at $t = 1$, ϕ is a parametric or semi-parametric function (e.g. polynomial basis expansions, splines, sparse regressions, or neural networks), and θ is calibrated on a training set of scenarios that are fully valued via nested simulation.

The economic rationale is straightforward: while the brute-force nested Monte Carlo approach requires solving a costly conditional valuation problem for each outer scenario, many of these valuations share similar structural dependence on the underlying state variables. If the mapping $X_1 \mapsto L_1$ is sufficiently smooth, it can be learned from a relatively small but carefully designed training set, and then evaluated cheaply across a much larger outer sample.

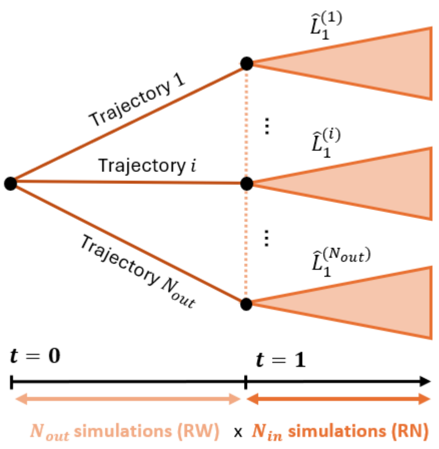

FIGURE 22: *Nested Monte Carlo method.*

In this way, the expensive inner simulation is performed only during calibration, rather than during every production run of the SCR.

Speed and scalability are the main benefits. Once trained, the proxy function ϕ can be evaluated almost instantaneously across a large set of outer scenarios, reducing computational cost from $\mathcal{O}(N_{\text{out}} \times N_{\text{in}})$ to approximately $\mathcal{O}(N_{\text{train}} \times N_{\text{in}} + N_{\text{out}})$, where N_{train} is the size of the calibration set. This enables frequent recalculations, sensitivity analyses, stress testing, and broad what-if coverage within practical time windows. In large life portfolios, this reduction in runtime is often the primary motivation for adopting proxy techniques.

The efficiency gain comes at the cost of introducing approximation error. Proxy methods concentrate model risk in regions where training density is low and they face extrapolation risk when outer scenarios fall outside the domain well represented by the calibration set. Their performance depends strongly on feature selection, basis design, and regularization choices, which introduces an additional layer of model risk and requires a robust governance framework, including careful calibration design, monitoring of parameter stability, periodic re-training, out-of-sample validation, and explicit assessment of tail accuracy.

A key practical challenge concerns the specification of the proxy function itself. Although convergence of LSMC regressions holds for fairly general families of basis functions, numerical performance can vary substantially depending on the chosen representation, the number and structure of regressors, and the dimensionality of the state space [11]. In solvency applications, where the goal is often to approximate the full distribution rather than only conditional expectations, the choice of basis becomes even more delicate and may require significant tuning. Furthermore, regression-based proxy approaches are exposed to the classical curse of dimensionality: as the dimension of the risk-factor vector increases, the number of basis terms required to maintain accuracy typically grows rapidly, leading to higher computational cost, instability, and potential overfitting issues [1]. This limitation is particularly relevant in ALM and capital models, where the state space can be high-dimensional and non-Markovian, making low-dimensional proxy specifications unable to fully capture the relevant dynamics and limiting scalability with respect to dimensionality.

Conceptually, the core trade-off is that simulation error (which is statistical and can be quantified via confidence intervals) is replaced by approximation error (which is structural and harder to bound rigorously). In particular, the impact of proxy misspecification on extreme quantiles is difficult to characterize with transparent probabilistic guarantees, making validation in a regulatory context both technically and operationally demanding. In practice, beyond the basic LSMC methodology, corrective and stabilization approaches, as well as advanced techniques for calibration, automatic selection of proxy structure, and computational optimization (see Moody's White Paper, [5]), can be introduced to mitigate these issues; however, the inherent approximation errors and limitations in extreme scenarios remain, requiring ongoing monitoring and robust model governance.

Multilevel Monte Carlo

From Nested Simulation to Multilevel Methods

As explained in Section *The Structure of SCR in an Internal Model*, under an Internal Model, the Solvency Capital Requirement is defined as:

$$SCR = VaR_{0.995}(\Delta BOF),$$

$$\Delta BOF = BOF_0 - BOF_1.$$

Since the VaR is a quantile, it is convenient to work with the associated exceedance probability. For any threshold $x \in \mathbb{R}$ define:

$$\begin{aligned} I(x) &:= \mathbb{P}(\Delta BOF > x) \\ &= \mathbb{E}^{\mathbb{P}}\left[\mathbf{1}_{\{\Delta BOF > x\}}\right]. \end{aligned} \tag{53}$$

Then SCR can be characterised as the smallest x such that $I(x) \leq 0.005$.
The nested structure arises because evaluating ΔBOF at time $t = 1$ requires a market-consistent liability value, which is a conditional risk-neutral expectation. Writing:

$$L_1 = \mathbb{E}^{\mathbb{Q}}[Y \mid \mathcal{F}_1],$$

with Y the discounted future cash flows under $\mathbb{Q}$, each outer scenario determines a value of L_1 only through an inner Monte Carlo estimation of this conditional expectation. Hence, estimating $I(x)$ leads to a nested Monte Carlo (NMC) procedure with computational cost proportional to $J \times K$, where J is the number of outer scenarios and K the number of inner simulations per scenario.
For tail quantities driven by discontinuous indicators, crude nested Monte Carlo exhibits an unfavourable accuracy–cost trade-off; in particular, achieving an RMSE of order ε typically entails a complexity of order $\mathcal{O}(\varepsilon^{-3})$, which rapidly becomes computationally prohibitive in realistic insurance applications (see e.g. [10, 3]). This computational barrier motivates replacing a single uniformly high-accuracy nested estimator with a multilevel construction. The MLMC framework has recently been advocated for SCR computation in insurance contexts [2], where it is shown to deliver substantial runtime reductions while preserving a fully simulation-based methodology.

General Principle of Multilevel Monte Carlo

MLMC replaces a single approximation with a hierarchy of increasingly accurate (and increasingly expensive) approximations, combined through a telescoping sum. In our setting, the quantity of interest is the indicator in (53), which we write as:

$$P(x) := \mathbf{1}_{\{\Delta BOF > x\}}.$$

The dependence on the inner valuation is implicit through $L_1 = \mathbb{E}^{\mathbb{Q}}[Y \mid \mathcal{F}_1]$.
Let $(P_\ell)_{\ell=0,\dots,L}$ be a sequence of approximations of P with increasing accuracy, indexed by a simulation level ℓ. MLMC relies on the the telescopic decomposition:

$$\mathbb{E}[P_L(x)] = \mathbb{E}[P_0(x)] + \sum_{\ell=1}^{L} \mathbb{E}\left[P_\ell(x) - P_{\ell-1}(x)\right].$$

Each term is estimated by Monte Carlo with sample size J_ℓ:

$$\widehat{I}_{\text{MLMC}}(x) = \frac{1}{J_0}\sum_{j=1}^{J_0} P_0^{(j)}(x) + \\ + \sum_{\ell=1}^{L}\frac{1}{J_\ell}\sum_{j=1}^{J_\ell}\Big(P_\ell^{(j)}(x) - P_{\ell-1}^{(j)}(x)\Big).$$

The essential mechanism is the following:

- Low levels (ℓ small) are computationally cheap but biased. They are simulated many times to reduce variance.
- High levels (ℓ large) are accurate but expensive. They are simulated only a few times to correct the bias.
- Strong coupling between P_ℓ and $P_{\ell-1}$ ensures that the variance of $P_\ell - P_{\ell-1}$ decreases with ℓ.

Under suitable regularity conditions, this strategy reduces the overall computational complexity from $\mathcal{O}(\varepsilon^{-3})$ for crude nested Monte Carlo to $\mathcal{O}(\varepsilon^{-2})$, which corresponds to the optimal Monte Carlo rate [7].

MLMC in the Nested SCR Framework

In the nested SCR setting, the levels typically correspond to different numbers of simulations J_ℓ and K_ℓ, often chosen in geometric progression:

$$J_\ell = J_0 2^{-\ell}, \quad K_\ell = K_0\, 2^\ell, \qquad \ell = 0, \dots, L.$$

At level ℓ, the conditional expectation $\mathbb{E}^{\mathbb{Q}}[Y \mid \mathcal{F}_1]$ is approximated by a Monte Carlo estimator with K_ℓ samples. The MLMC estimator then combines corrections across levels so that:

- The bias ($\mathbb{E}[P_L] - P$) induced by approximating the inner expectation is controlled by the maximum level L.
- The variance is reduced by allocating more outer samples J_ℓ to cheaper levels. In particular, the variance of the estimator is:

$$Var(P_L) = Var(P_0) + \sum_{l=1}^{L} Var(P_l - P_{l-1}).$$

This multilevel allocation results in a reduction of computational burden while preserving a fully simulation-based approach. Importantly, unlike proxy techniques such as Least Squares Monte Carlo or Replicating Portfolios, MLMC does not rely on parametric approximations of the response surface. This feature significantly simplifies model validation in an internal model context, as the methodology remains structurally close to full nested simulation.

Adaptive Refinements for Quantile Estimation

In the specific case of SCR estimation, the quantity of interest is driven by the indicator $g(u) = \mathbf{1}_{\{u \geq x\}}$, which is discontinuous at the loss threshold x. This lack of smoothness deteriorates the theoretical complexity of the standard MLMC estimator compared to the smooth-function case.
Giles and Haji-Ali (2019) [8] introduce adaptive allocation strategies in which the number of inner simulations depends on the distance of the conditional expectation from the loss threshold.
Intuitively, outer scenarios for which the estimated loss is far from the threshold require only a coarse inner estimate, while scenarios close to the threshold demand higher precision to avoid misclassification. This stochastic allocation of inner samples further improves efficiency and leads to near-optimal complexity of order:

$$\mathcal{O}\left(\varepsilon^{-2}\log(\varepsilon)^2\right).$$

Such adaptive MLMC schemes are particularly well suited for rare-event estimation and tail quantile computation, making them highly relevant for internal model SCR calculations in insurance.

Application to a Participating Life Product with Minimum Guarantee

This section introduces a representative toy example to illustrate the application of the Multilevel Monte Carlo (MLMC) methodology to the computation of the Solvency Capital Requirement (SCR).

Product Outline

Consider an index-linked life insurance contract with a minimum guaranteed benefit. The policy pays a benefit either at contractual maturity T or at the end of the policy year of death of the insured, denoted by t^* (discrete time). The payoff structure embeds both financial and biometric risk components and can be interpreted as a European put-type guarantee written on an underlying equity index.
Formally, let $(S_t)_{t\geq 0}$ denote the value of the reference equity index under a risk-neutral measure, and let K be the guaranteed capital. The value at time 0 is given by

$$V_0 = \mathbb{E}^{\mathbb{Q}}\left[e^{-r\,(T\wedge t^*)}\,(K - S_{T\wedge t^*})_+\right],$$

where r is the risk-free interest rate and $T \wedge t^*$ represents the minimum between maturity and the end of year of death. The contract therefore terminates at the earlier of death or maturity, and the guarantee ensures that the policyholder (or beneficiaries) receives at least K at termination. Economically, this structure resembles a put option with random maturity driven by the lifetime of the insured.
Despite its apparent simplicity, this contract captures the essential features of a broad class of life insurance products commonly observed in practice. First, it incorporates market risk through the stochastic dynamics of the underlying index S_t, typically modeled as a diffusion process (e.g., geometric Brownian motion) or within a more general risk-factor framework. Second, it embeds mortality risk via the random time of death t^*, which interacts with financial risk by inducing a stochastic and path-dependent effective maturity. Third, the presence of the minimum guarantee introduces a non-linear payoff profile, making the liability sensitive to downside equity movements and thereby generating significant tail risk, which is central in SCR calculations under Solvency II.
From a risk management perspective, this stylized product provides a natural laboratory for studying the joint impact of financial shocks and biometric uncertainty on the distribution of Own Funds. It also highlights the computational challenges associated with nested simulation approaches traditionally used for SCR estimation, particularly when guarantees and stochastic termination times are involved. For these reasons, it represents a parsimonious yet sufficiently rich benchmark for assessing the efficiency gains delivered by MLMC techniques in an insurance context.

SCR Framework and Numerical Implementation

In order to compute the Solvency Capital Requirement (SCR), a one-year risk horizon is adopted in accordance with the standard Solvency II framework. In particular, the relevant risk factors are simulated up to time one under the real-world probability measure $\mathbb{P}$. The two sources of uncertainty considered in this setting are: (i) the financial risk factor, represented by the underlying index (S_t), and (ii) the biometric risk factor, represented by the survival status of the insured, which are assumed to be independent.
More precisely, joint scenarios for $(S_1, \mathbf{1}_{\{1<t^*\}})$ are generated under $\mathbb{P}$. If the insured dies within the first year, the contract terminates and the liability at time one is fully determined by the payoff already realized. Conversely, if the insured survives the first year, the SCR requires the evaluation of the liability value at time one, denoted by V_1, conditional on the realized

market state S_1. In this case, V_1 corresponds to the risk-neutral value (under $\mathbb{Q}$) of the remaining cash flows from 1 to $T \wedge t^*$, given survival up to the first year.
Conditionally on survival and on S_1, the continuation value can be written as

$$V_1 = \mathbb{E}^{\mathbb{Q}} \left[e^{-r((T\wedge t^*)-1)} \left(K - S_{T\wedge t^*}\right)_+ \middle| \mathcal{F}_1 \right],$$

where the expectation is taken under the risk-neutral measure $\mathbb{Q}$, in accordance with market-consistent valuation principles and $\mathcal{F}_1$ represents the sigma-algebra up to time one.
To assess the numerical performance of different estimation techniques, the value V_1 is computed using several alternative approaches:

- **Closed-form benchmark.** Owing to the simplified structure of the product and assuming a geometric Brownian motion dynamics for S_t, a closed-form pricing formula can be derived. This value is used as a benchmark to assess the convergence properties and bias of the simulation-based estimators.
- **Nested Monte Carlo (NMC).** For each outer real-world scenario up to time one, an inner Monte Carlo simulation under $\mathbb{Q}$ is performed to estimate the conditional expectation defining V_1. While conceptually straightforward, this approach is computationally expensive due to the nested structure.
- **Multilevel Monte Carlo (MLMC).** In order to reduce computational cost, the MLMC estimator is implemented. The telescopic decomposition of expectations across levels allows to control bias and variance simultaneously, achieving a significant reduction in complexity compared to standard nested Monte Carlo.

This setup mirrors the practical SCR computation problem: a first-stage projection of risk factors over one year under $\mathbb{P}$, followed by a second-stage market-consistent valuation under $\mathbb{Q}$ of the remaining liabilities. The toy example therefore provides a controlled environment in which the trade-off between accuracy and computational complexity of different Monte Carlo methodologies can be rigorously assessed.

Numerical Results

The financial market is modeled through a geometric Brownian motion under the real-world probability measure $\mathbb{P}$:

$$dS_t = \mu S_t\, dt + \sigma S_t\, dW_t^{\mathbb{P}},$$

while under the risk-neutral measure $\mathbb{Q}$ the drift is replaced by the risk-free rate r.
The baseline parameters used in the numerical experiments are:

$$S_0 = 100, \quad K = 100, \quad \sigma = 0.30,$$

$$\mu = 0.05, \quad r = 0.02, \quad T = 20.$$

The initial age of the insured is set to 50, and survival probabilities are derived from Italian population life tables published by ISTAT (latest available year). Financial and biometric risks are assumed to be independent.
Even though a closed-form pricing formula is available for the continuation value, the Solvency II framework requires simulation of the one-year distribution under the real-world measure $\mathbb{P}$.
The benchmark procedure is therefore structured as follows:

1. Simulate S_1 under $\mathbb{P}$;
2. Simulate the survival status over $[0, 1]$;
3. If death occurs within the first year, the liability is fully realized.
4. If survival occurs, compute:

$$V_1 = \mathbb{E}^{\mathbb{Q}} \left[e^{-r((T\wedge t^*)-1)} \left(K - S_{T\wedge t^*}\right)_+ \middle| \mathcal{F}_1 \right].$$

Considering the independence between the mortality and the financial risk, V_1 can

	Classical Monte Carlo			Multilevel Monte Carlo		
N_{it}	J	K	Bias (bps)	J_0	K_0	Bias (bps)
64,000,000	32,000	2,000	6.08	16,000	1,000	5.48
128,000,000	64,000	2,000	5.77	32,000	1,000	0.46
256,000,000	128,000	2,000	1.08	64,000	1,000	0.95
512,000,000	256,000	2,000	5.61	128,000	1,000	1.73

TABLE 8: *Example results of the MLMC and classical Nested MC algorithms.*

be computed via a closed-form formula. Hence, the benchmark still requires outer real-world simulation but avoids inner simulation by replacing it with the analytical formula. This benchmark is used to assess bias and convergence of simulation-based estimators.
As explained in Section *MLMC in the Nested SCR Framework*, the computational effort for each level is defined through the following geometric allocation:

$$J_\ell = J_0 2^{-\ell}, \quad K_\ell = K_0 2^\ell, \quad \ell = 0, \dots, L.$$

This construction ensures a progressive refinement of the inner conditional expectation across levels while simultaneously reducing the number of outer simulations. The geometric structure not only stabilizes the variance of the level differences, but also allows for a convenient expression of the total computational budget. Indeed, since each level requires $J_\ell \times K_\ell = J_0 \times K_0$ iterations, the total number of iterations of the MLMC algorithm is $N_{\text{it}} = (L+1) \times J_0 \times K_0$.
In our numerical experiment, we set:

$$L = 3, \qquad J_0 \in \{2000 \cdot 2^k : k = 0, \dots, 8\},$$

$$K_0 \in \{1000, 2000, 3000, 4000, 5000, 7500, 10000\}.$$

In order to ensure a coherent comparison with the classical Nested Monte Carlo estimator, the parameters (J, K) of the nested method are selected so that the overall computational effort coincides with that of MLMC.

More precisely,

$$J \times K = (L+1) \times J_0 \times K_0.$$

In this way, both approaches are compared under identical total iteration counts.
For illustrative purposes, Table 8 reports a small subset of representative configurations. The table provides an example of how the computational budget is allocated and the resulting biases (expressed in basis points with respect to the analytical benchmark).
A graphical comparison of the two estimators is now presented in order to better highlight their convergence behavior and stability properties.
Figure 23 displays the bias as a function of the total computational budget N_{it} for both MLMC and classical Nested Monte Carlo. This two-dimensional representation isolates the effect of the overall computational effort and allows to directly assess the relative speed of convergence of the two estimators.
Several patterns emerge from the comparison. For relatively low computational budgets, both estimators exhibit non-negligible bias, with fluctuations driven by the variance of the outer scenarios and by the accuracy of the inner conditional expectation estimates. In this region, the classical nested approach may occasionally display slightly smaller bias due to its uniform allocation of computational effort across all scenarios. However, as the total number of iterations increases, the MLMC estimator exhibits a

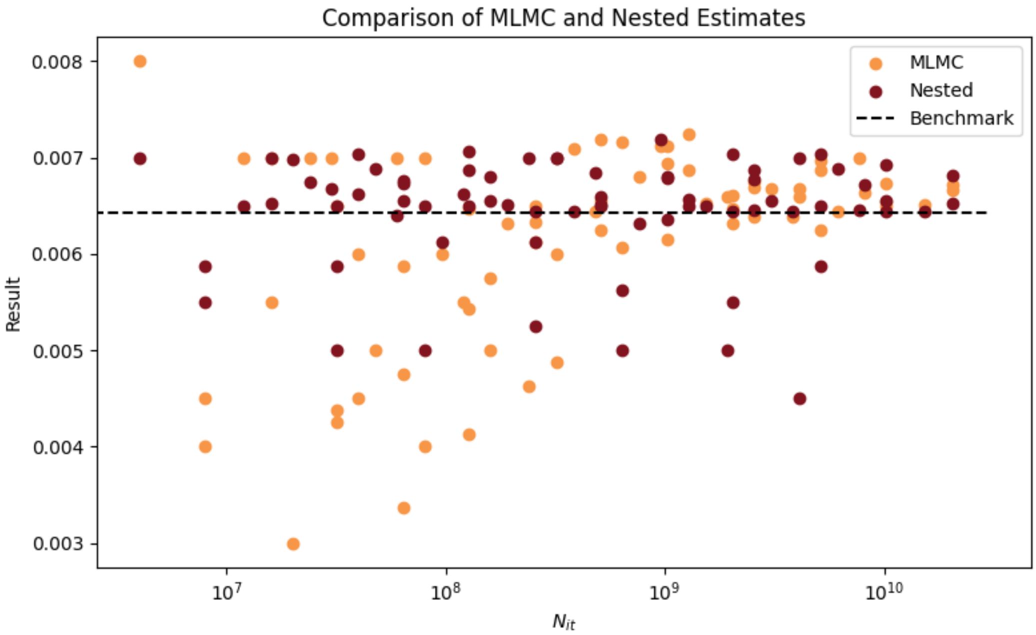

FIGURE 23: Bias as a function of the total computational budget N_{it} for MLMC and classical Nested Monte Carlo. For large N_{it}, the MLMC algorithm remains closer to the benchmark value, while the classical Nested Monte Carlo still shows significant error.

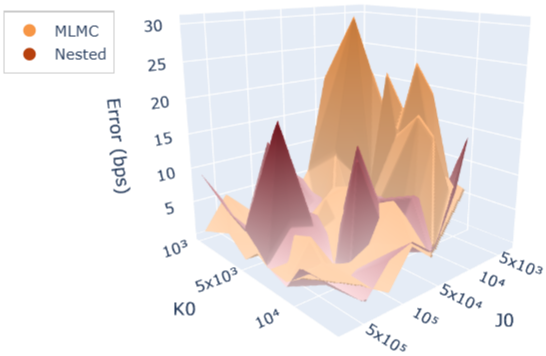

FIGURE 24: Bias surface (in basis points) as a function of outer (J_0) and inner (K_0) simulation parameters for MLMC and classical Nested Monte Carlo.

more stable convergence pattern. The bias decreases more regularly with respect to N_{it}, reflecting the variance reduction mechanism induced by the telescopic decomposition across levels. In contrast, the classical nested estimator shows less regular behavior, with occasional spikes in the bias even at higher computational budgets. This instability is primarily driven by the inefficient allocation of inner simulations across all outer scenarios, regardless of their contribution to the overall variance.

Figure 24, instead, provides a three-dimensional visualization of the bias as a function of the outer and inner simulation parameters. This representation offers additional insight into how the allocation of computational resources between outer and inner simulations influences the accuracy and robustness of the estimators, thereby complementing the analysis based solely on the total budget.

For low values of J_0 (or J), corresponding to a limited number of outer real-world scenarios, the MLMC estimator exhibits higher bias. This reflects the fact that insufficient sampling of the one-year loss distribution negatively impacts the accuracy of tail esti-

mation, which is central for SCR computation.
However, as both the number of outer scenarios and the inner simulation effort increase, the MLMC estimator demonstrates a smoother and more stable convergence surface. In particular, the bias does not display the pronounced peaks observed in the classical nested case. This highlights the efficiency of the multilevel allocation, which concentrates computational effort where it contributes most to variance reduction.
Overall, the numerical evidence confirms that, while MLMC may require a minimal scale to outperform the classical nested approach, it provides superior scalability and robustness as computational resources increase. This property is particularly relevant in realistic insurance applications, where high-dimensional portfolios and long maturities make fully nested simulation computationally prohibitive.

Conclusions

Internal Model SCR computation under Solvency II requires robust estimation of tail risk based on one-year changes in BOF under market-consistent valuation at the projection horizon. Although fully nested Monte Carlo provides a conceptually straightforward and model-consistent benchmark, its computational complexity (scaling with $N_{\text{out}} \times N_{\text{in}}$) together with the intrinsic difficulty of tail estimation, often limits its practical applicability at industrial scale. Proxy-based approaches reduce runtime but shift the burden toward model specification, governance, and validation, while introducing approximation risk that can be particularly material in the SCR tail.
Multilevel Monte Carlo provides a coherent alternative that preserves the nested valuation structure while improving efficiency through variance reduction and level coupling. By exploiting a telescoping decomposition across simulation resolutions, MLMC achieves faster convergence compared to standard nested simulation, enabling accurate SCR estimation with significantly lower computational cost. The adaptive extension further strengthens this framework by reallocating computational effort, increasing precision where it matters most while keeping additional costs contained.
Overall, the proposed methodology combines statistical robustness, scalability, and transparency, offering a practical pathway toward industrial SCR computation without relying on proxy approximations. A particularly promising extension is the use of multi-index Monte Carlo methods, where refinement is performed simultaneously along several discretization dimensions rather than only across inner simulation levels. In insurance applications, these dimensions may include time discretization of stochastic processes or numerical approximations embedded in path-dependent liability valuation. By exploiting mixed-difference estimators across multiple axes, multi-index schemes can mitigate the exponential growth in computational effort that typically arises when several sources of discretization error interact. This direction, especially relevant for portfolios with strong path dependence, will be investigated in detail in forthcoming work, where a multi-index extension of the proposed framework will be introduced and analysed.

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SFDR 2.0 and EU Taxonomy in Asset Management Field

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The authors present an analysis of the European regulatory framework governing sustainable finance, with particular reference to the implications for asset management companies. The contribution examines the three fundamental legislative components of the European Union's ESG architecture: the Sustainable Finance Disclosure Regulation (SFDR), which introduces harmonised transparency obligations at both entity and product level; the EU Taxonomy Regulation, which establishes a classification system for environmentally sustainable economic activities; and the Corporate Sustainability Reporting Directive (CSRD), which governs corporate sustainability reporting through the European Sustainability Reporting Standards (ESRS).
Particular attention is devoted to the proposed revision commonly referred to as SFDR 2.0, through which the European Commission intends to introduce targeted adjustments to the existing regime, including a revision of product categorisation and a rationalisation of transparency requirements. This proposal marks a significant evolution towards a more structured system of financial product classification, aimed at improving comparability and reducing greenwashing risks. For the asset management industry, this shift carries significant strategic implications, requiring a realignment of portfolio construction and more robust ESG data governance. The authors conclude that the reform represents both a regulatory challenge and a strategic opportunity for firms capable of credibly integrating sustainability considerations into their investment processes.

In the field of environmental, social and governance (ESG) regulation, the European Commission has progressively developed a comprehensive legislative framework, subsequently refined through a series of amendments and delegated acts, with the stated objective of supporting the transition towards a modern, resource-efficient and competitive economy. This regulatory architecture is embedded within the broader policy strategy set out in the *European Green Deal*, which identifies sustainable finance as a structural instrument for aligning private capital allocation with the Union's climate and environmental objectives.
In this context, the European legislature has sought to address information asymmetries, enhance the comparability and reliability of sustainability-related data, and mitigate the risks of greenwashing, thereby reinforcing market integrity and investor confidence.
At present, the principal regulatory cornerstones of the ESG framework applicable to asset management companies are represented by three interrelated legislative instruments.
First, Regulation (EU) 2019/2088 of the European Parliament and of the Council on sustainability-related disclosures in the financial services sector (better known as the *Sustainable Finance Disclosure Regulation*, hereinafter **SFDR**) establishes harmonised transparency requirements for financial market participants and financial advisers. The SFDR introduces obligations at both entity and product level, requiring disclosure of policies concerning the integration of sustainability risks, the consideration of principal adverse impacts on sustainability factors, and the categorisation of financial products according to their sustainability characteristics. The Regulation is designed to ensure that end investors receive consistent, comparable and decision-useful information, thereby facilitating informed investment decisions.
Secondly, Regulation (EU) 2020/852 on the establishment of a framework to facilitate sustainable investment (better known as the *EU Taxonomy Regulation*, hereinafter the **EU Taxonomy**) creates a legally binding classification system for environmentally sustainable economic activities. By defining detailed technical screening criteria for determining whether an activity substantially contributes to one or more environmental objectives and does not significantly harm the others, the EU Taxonomy seeks to provide a common reference framework for market participants and to counteract the

marketing of financial products as sustainable in the absence of objective and verifiable benchmarks. The EU Taxonomy is expressly integrated into the SFDR disclosure regime, which requires financial market participants to report on the degree of Taxonomy alignment of certain financial products.
Thirdly, Directive (EU) 2022/2464, which amends Regulation (EU) N. 537/2014 and Directives 2004/109/EC, 2006/43/EC and 2013/34/EU as regards corporate sustainability reporting (better known as the *Corporate Sustainability Reporting Directive*, hereinafter the **CSRD**), significantly expands the scope, depth and assurance requirements of non-financial reporting. The sustainability reporting standards applicable to undertakings subject to the CSRD, namely the European Sustainability Reporting Standards (**ESRS**), were adopted by the Commission through Delegated Regulation (EU) 2023/2772, thereby specifying the structure and substantive content of the information to be disclosed.
The enhanced availability of reliable, comparable and standardised sustainability data across the value chain consequently supports the effectiveness of financial market disclosures and the proper functioning of the sustainable finance framework as a whole.
Tables 9 and 10 set out, for each of the aforementioned regulatory instruments, the principal differences with respect to the scope of entities covered, the subject matter, and the entry into force. Against this background, the present article retraces the regulatory evolution of the aforementioned legislative foundations, analysing their interaction and progressive refinement over time. Particular attention is dedicated to the legislative proposal commonly referred to as **SFDR 2.0**, through which the European Commission is considering targeted adjustments to the existing disclosure regime, including potential revisions to product categorisation, the clarification of key concepts and the rationalisation of transparency requirements.
The analysis places these developments within the broader objective of recalibrating the Union's sustainable finance architecture so as to more effectively redirect capital flows towards sustainable investments, while enhancing legal certainty and ensuring proportionality within the overall regulatory system.

ESG Regulation: the Current Regulatory Framework and its Evolution

The following paragraph outlines the regulatory evolution of the reference legislative instruments cited in the introduction, together with the sector-specific frameworks applicable to asset management companies that have been amended over time to incorporate ESG considerations.
In addition to SFDR, EU Taxonomy and CSRD, the integration of sustainability factors has also affected the regulatory regimes governing investment firms and management companies under Directive 2014/65/EU (**MiFID II**), Directive 2009/65/EC (**UCITS**) and Directive 2011/61/EU (**AIFMD**).
Tables 17 and 18 present, for each of the aforementioned legislative measures, the principal differences with respect to the scope of entities covered, the subject matter, and the entry into force.
Through subsequent delegated acts and amendments, these frameworks have introduced requirements relating to the integration of sustainability risks into organisational arrangements, product governance, suitability assessments and investment decision-making processes.
The resulting regulatory trajectory reflects a progressive alignment of both horizontal disclosure obligations and sectoral prudential rules with the objectives of the Union's sustainable finance agenda.

Legislation	Scope of Entities	Subject Matter	Entry into Force
SFDR Regulation	**Asset Managers** • UCITS management companies (investment management companies, SICAVs) *Art. 2(1)(a);* • Alternative Investment Fund Managers (AIFMs) *Art. 2(1)(b);* • Managers of EuVECA, EuSEF, ELTIF *Art. 2(1)(e)(f)(g);* • Financial advisers (natural and legal persons) *Art. 2(11).* **Banks / Investment Firms** • Investment firms providing individual portfolio management services *Art. 2(1)(i);* • Credit institutions providing individual portfolio management services *Art. 2(1)(j).* **Insurance Undertakings** • Insurance undertakings making available insurance-based investment products (IBIPs) *Art. 2(1)(c).* **Pension Institutions** • Institutions for Occupational Retirement Provision (IORPs) *Art. 2(1)(d).* **All** **Principal Adverse Impacts (PAI) threshold**, *Art. 4(1)* : • ≥ 500 employees → mandatory PAI statement *[Art. 4(1)(a)];* • < 500 employees → comply or explain *[Art. 4(1)(b)].*	SUSTAINABILITY-RELATED DISCLOSURE OBLIGATIONS • *Art. 3* Sustainability risk integration policies (website disclosure); • *Art. 4* Principal Adverse Impacts statement at entity level; • *Art. 5* Remuneration policies consistent with sustainability risk integration; • *Art. 6* Integration of sustainability risks in pre-contractual disclosures; • *Art. 7* Transparency on PAI at financial product level; • *Art. 8* Products promoting environmental or social characteristics ("light green"); • *Art. 9* Products with a sustainable investment objective ("dark green"); • *Art. 10* Website transparency obligations; • *Art. 11* Periodic (annual) reporting.	**Entry into force** *29.12.2019* **Application of main obligations** *10.03.2021* **Level 2 RTS** *Del. Reg. 2022/1288* *01.01.2023* **Italy** *Direct application*
EU Taxonomy Regulation	**Asset Managers** • Financial market participants within the meaning of SFDR making available financial products with environmental objectives *Art. 5;* • Financial market participants making available other financial products *Art. 6;* • Financial market participants making available financial products with social objectives *Art. 7.* **Corporate** • Large public-interest entities subject to non-financial reporting obligations: disclosure of taxonomy-aligned turnover, CapEx and OpEx *Art. 8.*	TAXONOMY CRITERIA AND DISCLOSURE OBLIGATIONS • *Art. 3* Four cumulative criteria: (i) substantial contribution to at least one objective; (ii) Do No Significant Harm (DNSH) to the other five; (iii) minimum social safeguards; (iv) compliance with Technical Screening Criteria (TSC); • *Art. 4* Obligation of Member States and the EU to refer to the Taxonomy when setting requirements for market operators and public instruments; • *Art. 5* Mandatory transparency for financial products with environmental objectives (Art. 9 SFDR); • *Art. 6* Transparency for financial products not promoting environmental objectives; • *Art. 7* Transparency for financial products with social objectives;	**Entry into force** *12.07. 2020* **Climate objectives [Art. 9(a)(b)]** *01.01.2022* Technical Screening Criteria: Del. Reg. (EU) 2021/2139 **Other 4 objectives [Art. 9(c)–(f)]** 01.01.2023 Technical Screening Criteria: Del. Reg. (EU) 2023/2486 **Italy** *Direct application*

TABLE 9: *Comparison between SFDR, EU Taxonomy and CSRD [Table 1/2].*

Legislation	Scope of Entities	Subject Matter	Entry into Force
EU Taxonomy Regulation		**Taxonomy criteria and disclosure obligations** • *Art. 8* Corporate disclosure: % of taxonomy-aligned turnover, CapEx and OpEx; detailed rules in Del. Reg. (EU) 2021/2178; • *Art. 9* Six environmental objectives: climate change mitigation, adaptation, water, circular economy, pollution prevention, biodiversity.	
CSRD Directive	**Large Undertakings** • Large EU undertakings meeting ≥ 2 of 3 criteria: >250 employees / >€40M net turnover / >€20M total assets *Art. 1 (amending Art. 19a Dir. 2013/34/EU)*; • Public-interest entities (PIEs) *Art. 1 (amending Art. 19a)*; • Parent undertakings of large groups *Art. 1 (amending Art. 29a)*; • Non-EU undertakings with net turnover >€150M in the EU and at least one large subsidiary or listed branch *Art. 1 (new Art. 40a)*. **Listed SMEs** • SMEs listed on EU regulated markets, except micro-undertakings; opt-out available until financial year 2028 *Art. 1 (amending Art. 29a)*.	**Sustainability reporting** • *Art. 1 (amending Art. 19a)* Obligation to include sustainability reporting in the management report; • *Art. 1 (amending Art. 29a)* Consolidated sustainability reporting for parent undertakings; • *Art. 1 (new Art. 40a)* Reporting for non-EU undertakings; • Double materiality: impact materiality (effects of the undertaking on environment and society) + financial materiality (ESG effects on performance); • European Sustainability Reporting Standards (ESRS) adopted by Del. Reg. (EU) 2023/2772; • Mandatory limited assurance by an external auditor; • iXBRL format for digital interoperability; • *Art. 1 (amending Art. 33)* Responsibility of management and supervisory bodies.	**Entry into force** *05.01.2023* **Phased application (financial years)** ***FY 2024***: large PIEs >500 employees (former NFRD scope) ***FY 2025:*** other large undertakings ***FY 2026:*** listed SMEs (opt-out until FY 2028) ***FY 2028:*** non-EU undertakings (Art. 40a)) **Italy** Legislative Decree 6 September 2024, N. 125 Italian Official Journal N. 212, 10.09.2024 – in force 25.09.2024

TABLE 10: *Comparison between SFDR, EU Taxonomy and CSRD [Table 2/2].*

Corporate Sustainability Reporting Directive

On 14 December 2022, the CSRD was published in the Official Journal of the European Union with the aim of enhancing investors' ability to understand so-called non-financial information.
The measure responded to a marked increase in demand for sustainability-related disclosures, particularly with regard to climate-related financial risks. This regulatory intervention was considered necessary in light of the rapid expansion of investment products explicitly designed to comply with specific sustainability principles, which in turn required more reliable, comparable and standardised corporate reporting.
Within the Italian legal order, the CSRD was implemented by Legislative Decree N. 125 of 6 September 2024, published in the Official Gazette of the Italian Republic N. 212 of 10 September 2024, thereby incorporating the European sustainability reporting framework into domestic company law.
On 31 July 2023, the European Sustainability Reporting Standards (ESRS) were adopted. These standards specify the environmental, social and governance matters that undertakings falling within the scope of the CSRD are required to disclose.
The ESRS therefore represent the technical and operational mechanism through which the objectives established by the Directive are implemented, translating general reporting obligations into detailed and structured disclosure requirements.
The European legislator also took into account requests from the business community for greater interoperability between European sustainability reporting standards and global standards [5]. In this respect, steps have been taken to promote alignment between the ESRS and the standards issued by the International Sustainability Standards Board (ISSB), enabling undertakings to pursue compliance with both frameworks in a coordinated manner.

Notwithstanding its objectives, the scope initially envisaged by the CSRD was regarded as excessively burdensome for smaller undertakings. In the course of 2025, a process of simplification was initiated through the so-called *Omnibus package*, which was ratified on 24 February 2026 by the European Council and now pending publication in the Official Journal of the European Union, primarily affecting the range of entities subject to the Directive. The revised approach concentrates sustainability reporting obligations on larger undertakings and seeks to ensure that such obligations do not impose disproportionate burdens on smaller companies within their value chains. At the same time, the reporting timetable was adjusted through the adoption of the so-called *Stop-the-Clock* Directive, which postponed by two years the application of CSRD reporting requirements for large undertakings not yet subject to the obligations and for listed small and medium-sized enterprises. In Italy, this postponement was transposed into national law by Law N. 118 of 8 August 2025, published in the Official Gazette of the Italian Republic N. 184 of 9 August 2025, in particular through Article 10, paragraph 1-bis.
For asset management companies, the ESG data required under the Sustainable Finance Disclosure Regulation (SFDR) is largely derived from the sustainability information disclosed by undertakings subject to the CSRD.
The Directive therefore plays a fundamental role in shaping investment decisions and disclosure practices within asset management, as the quality, scope and comparability of CSRD reporting directly influence both portfolio allocation strategies and the sustainability disclosures made at entity and product level.

European Sustainability Reporting Standards

Commission Delegated Regulation (EU) 2023/2772 of 31 July 2023, adopted by the

European Commission, formally adopts and incorporates the European Sustainability Reporting Standards (ESRS) into EU law, defining their content, structure and disclosure requirements. The ESRS Regulation clarifies that disclosures must address the undertaking's material impacts, risks and opportunities in relation to environmental, social and governance matters, where these are considered relevant following the materiality assessment.

The purpose of such disclosures is twofold: first, to enable users of the sustainability statement to understand the significant impacts of the undertaking on people and the environment; secondly, to provide insight into the material effects of sustainability matters on the undertaking's development, performance and financial position.

In this sense, the ESRS operate as the technical and methodological framework through which the sustainability reporting obligations introduced by the CSRD are rendered concrete and comparable across undertakings. The internal structure of the ESRS is articulated around three distinct but interrelated components.

The first consists of ***cross-cutting standards***, namely *ESRS 1 General requirements* and *ESRS 2 General disclosures*, which establish the overarching principles governing the preparation of the sustainability statement and set out general disclosure requirements applicable irrespective of sector or topic.

The second component comprises ***topical standards*** covering environmental, social and governance matters. These are organised into themes and sub-themes, and, where appropriate, further sub-sub-themes, in accordance with Application Requirement 16 of the Regulation, thereby ensuring a granular and systematic articulation of sustainability topics.

The third component is represented by ***sector-specific standards***, intended to address impacts, risks and opportunities that are likely to be material for all undertakings operating within a given sector and that are not covered, or not sufficiently addressed, by the thematic standards.

A defining feature of the ESRS framework is the formalisation of the principle of *double materiality*. Pursuant to paragraph 28 of the Regulation, a sustainability matter qualifies as material where it meets the criteria for impact materialitynamely, where the undertaking's activities have actual or potential significant impacts on people or the environmentor the criteria for financial materiality, that is, where sustainability matters generate or may generate material financial effects on the undertaking, or both. Undertakings are therefore required to conduct a structured materiality assessment encompassing both dimensions, and to base the scope of their disclosures on the outcome of that assessment.

The ESRS further prescribe the structure of the sustainability statement. This is composed of four main sections:

- *General information*, as required under ESRS 2;
- *Environmental information*, including in a clearly identifiable section the disclosures required under Article 8 of the EU Taxonomy;
- *Social information*; and
- *Governance information*.

Figure 25 provides a schematic representation of this structure, which is intended to promote consistency in presentation and facilitate comparability between undertakings.

General Information	General disclosures, including information provided under the Application Requirements of topical ESRS listed in ESRS 2 Appendix C
Environmental Information	▪ Disclosures pursuant to Article 8 of Taxonomy Regulation ▪ Climate change (ESRS E1) ▪ Pollution (ESRS E2) ▪ Water and marine resources (ESRS E3) ▪ Biodiversity and ecosystems (ESRS S4) ▪ Resource use and circular economy (ESRS E5)
Social Information	▪ Own workforce (ESRS S1) ▪ Workers in the value chain (ESRS S2) ▪ Affected communities (ESRS S3) ▪ Consumers and end-users (ESRS S4)
Governance Information	Business conduct (ESRS G1)

FIGURE 25: Structure of the ESRS Sustainability Statement in accordance with Appendix D of the ESRS Regulation.

Opinion of the European Central Bank

In February 2026, the European Central Bank, responding to an invitation extended by the European Commission, set out its formal opinion on the revision of the European Sustainability Reporting Standards proposed by EFRAG in December 2024 [4]. The ECB's assessment focused on a selected cluster of standards: ESRS 1 (*General Requirements*) and ESRS 2 (*General Disclosures*), together with, within the environmental disclosure framework, ESRS E1 (*Climate Change*) and ESRS E4 (*Biodiversity and Ecosystems*).

Among the changes put forward by EFRAG, the ECB expressed particular **appreciation for three developments**.

The first concerns the enhanced clarity introduced with respect to both the information to be reported, as captured by disclosure requirements (DRs), and the methodologies governing how such disclosures are to be made, as set out in the application requirements (ARs). The second relates to the clearer demarcation between mandatory application requirements (ARs) and non-mandatory guidance situated outside the body of the draft standards. The third pertains to the greater transparency afforded regarding the application of the materiality filter, which determines the scope of information that entities are required to disclose.

Alongside these positive assessments, **three areas of concern** were nonetheless highlighted by the ECB as requiring attention if the revision is to strike an appropriate balance between simplification and the preservation of the policy objectives embedded in the CSRD.

The first relates primarily to on the cumulative effect of the wide array of permanent reliefs, phased implementation provisions and carve-outs from reporting obligations that have been incorporated into the revised standards: taken together with the elimination of certain datapoints previously deemed essential, these measures are liable to reduce the overall stock of meaningful in-

formation available to users and to weaken the consistency of sustainability statements across different companies.
The second area of concern relates to interoperability with international benchmarks, and specifically with the frameworks developed by the IFRS and the ISSB: whilst the ECB acknowledged the steps taken in this direction, it noted that the revised proposals depart from those international benchmarks in certain respects, particularly through the inclusion of reliefs that go beyond what those frameworks foresee.
The third and final point concerns the adequacy of the revised standards for the specific disclosure needs of the financial sector, where the ECB considered that a number of issues remain insufficiently defined to allow for disclosures that would be genuinely informative for financial institutions and market participants.
The **revision of the ESRS**, unfolding simultaneously with the **review of the SFDR**, calls for a careful assessment of which disclosure requirements are to be removed. The proposal to eliminate from the SFDR the company-level disclosure obligations is driven by the need to remove duplications with respect to the provisions of the CSRD, on the assumption that the ESRS adequately cover sustainability disclosures at the level of the individual undertaking, whilst the SFDR would retain responsibility solely for product-level disclosures.
Given that the ESRS are sector-agnostic, the first set of standards did not incorporate adequately defined metrics for the value chain of the financial sector. By way of illustration, section ESRS E1-7, which addresses energy consumption and energy mix, is confined to the reporting entity's own operations and does not extend to value chain activities. It follows that the standards lack sufficiently defined metrics for the specific characteristics of the financial sector: the energy intensity of portfolios, for instance, was an explicitly prescribed datapoint under the SFDR yet finds no equivalent in the current architecture of the ESRS. This gap risks translating into a net reduction in the availability of information that is relevant to financial sector operators, at the very moment when the regulatory simplification exercise is seeking to eliminate redundancies between the two frameworks.

EU Taxonomy

The EU Taxonomy Regulation was adopted in response to the need to reorient capital flows towards sustainable investments in order to achieve sustainable and inclusive growth. To this end, the Regulation defines a common classification system through which economic activities may be considered as contributing to environmental objectives, with the prospect of subsequently extending the framework to encompass social objectives at a later stage.
The harmonisation of criteria for identifying the sustainability of an economic activity is likewise intended to support investors in the assessment of environmentally sustainable financial products, while reducing the risk of fragmentation within the internal market. Through the introduction of a shared Taxonomy, the Regulation enhances comparability, transparency and legal certainty across Member States. In this respect, it complements and integrates the transparency obligations relating to precontractual disclosures and periodic reports established under the SFDR, thereby ensuring consistency between the classification of environmentally sustainable activities and the sustainability-related information provided to investors. Where economic activities fall outside the scope of the EU Taxonomy, and financial market participants are not reasonably in a position to obtain the necessary information concerning their environmental sustainability, the Regulation permits reliance on estimates and reasoned assessments. In such circumstances, the assumptions, methodologies and conclusions underpinning the evaluation process must be clearly explained and appropriately disclosed, thereby ensuring transparency in

the absence of complete data.
The EU Taxonomy identifies six environmental objectives against which the sustainability of an economic activity is to be assessed. These are:

- Climate change mitigation[7];
- Climate change adaptation[8];
- The sustainable use and protection of water and marine resources;
- The transition to a circular economy;
- The pollution prevention and control; and
- The protection and restoration of biodiversity and ecosystems.

Article 3 of EU Taxonomy sets out the cumulative conditions that an economic activity must satisfy in order to qualify as environmentally sustainable. Specifically, the activity must:

- Contribute substantially to one or more of the above-mentioned environmental objectives;
- Not significantly harm any of the other environmental objectives;
- Be carried out in compliance with minimum safeguards;
- Comply with the relevant technical screening criteria established by delegated acts.

Following its publication in the Official Journal of the European Union on 22 June 2020, the EU Taxonomy Regulation was supplemented by a series of delegated acts adopted by the European Commission to establish the technical screening criteria applicable to specific economic activities. These delegated acts (e.g. the Climate Delegated Act) define in detail the conditions under which particular activities may be regarded as making a substantial contribution to the relevant environmental objectives and meeting the requirements laid down in the Regulation, thereby specifying the practical application of the Taxonomy framework.
Articles 5 and 6 of EU Taxonomy further regulate disclosure obligations in relation to environmentally sustainable investments and to financial products that promote environmental characteristics. In parallel, within the CSRD framework, non-financial undertakings are required to disclose (a) the proportion of their turnover derived from products or services associated with environmentally sustainable economic activities, and (b) the proportion of their capital expenditure and operating expenditure related to assets or processes associated with such activities. These disclosure requirements are particularly relevant for asset management companies, as they provide an objective benchmark of environmental sustainability for the purposes of transparency in financial products classified under Article 8 or Article 9 SFDR.

Sustainable Finance Disclosure Regulation

Following the entry into force of the SFDR Consob, through its supervisory notices, provided a non-exhaustive list of positive operational practices identified in the course of its supervisory activity, with a view to supporting intermediaries in achieving compliance with the regulatory framework. The first Consob Notice [2] focused on the adaptation to obligations concerning "sustainable finance" as regards the ESG profiles of intermediaries in the provision of investment services, as well as entity-level disclosure requirements. The second Consob Notice [3], by contrast, concentrated on compliance with certain key elements of the regulatory framework re-

[7]Namely contributing substantially to the stabilisation of greenhouse gas concentrations by avoiding or reducing emissions or enhancing their removal.
[8]That is, contributing substantially to reducing or preventing the adverse impacts of the current or expected future climate, or the risk of such impacts, on the activity itself or on people, nature or assets.

<table>
<tr><th>CONSOB Notice N. 1/24</th><th>CONSOB Notice N. 1/25</th></tr>
<tr><td>Sustainability Transparency under SFDR
• Disclosures must be clear, accurate, non-misleading and compliant with MiFID II and SFDR: easily accessible, visible, simple, concise and non-discriminatory.
• On the intermediary's website, sustainability disclosure at entity level must be identifiable and immediately accessible.
• Sustainability risk and PAIs must be integrated into product selection criteria, described in the relevant policies and clearly linked to the satisfaction of clients' ESG preferences.
• SFDR obligations must be complied with as regards templates, completion instructions and prescribed publication timelines.
• Financial Market Participants must publish in the "Sustainability Disclosure" section information on portfolio management under Arts. 8 and 9 SFDR and periodic disclosures under Art. 11 SFDR.
Assessment of Clients' Sustainability Preferences within the Suitability Test
• "Sustainability preferences" and the related concepts must be explained using plain language, free of technical terminology.
• The collection of information on sustainability preferences must be sufficiently granular, including elements not expressly required by regulation where deemed necessary.
• In the product mapping phase of the suitability test, sustainability characteristics must be analysed in depth: overly simplified solutions risk undermining the effective suitability of investments with respect to clients' preferences.
• If no product meets a client's sustainability preferences, the client must be able to temporarily adapt them, with the choice being recorded, without permanently modifying the preferences gathered via questionnaire.
• In retail advisory services, the suitability statement must confirm the positive outcome of the test also with regard to sustainability preferences and any adaptation chosen by the client.</td>
<td>Assessment of Clients' Sustainability Preferences within the Suitability Test
With regard to pre-contractual disclosure, Consob draws attention to disclosures relating to:
• The identification of sustainable objectives and environmental and/or social characteristics, as well as the indicators used to measure the degree to which they are achieved;
• With regard to sustainable investments, compliance with the "Do No Significant Harm" principle;
• The binding elements of the investment policy and the policy for assessing good governance.
With regard to the definition of clear and accurate information for end investors, Consob draws attention to the following aspects:
• Clear and precise identification of the characteristics promoted and/or objectives pursued;
• Identification of one or more indicators for assessing each of the characteristics promoted/objectives pursued;
• Representation of the assessment approach adopted to verify compliance with the DNSH principle and the safeguard clauses;
• Representation of the information on the strategy and verification of good governance (for the latter, in particular, the clear representation that such verification applies to all investments in the portfolio);
• Designation of benchmarks for Art. 9 products based on methodologies and criteria of a "sustainable" nature.
With regard to periodic disclosure (Annexes IV and V of Commission Delegated Regulation (EU) 2022/1288), Consob draws operators' attention to:
• The representation of the ex-post value of each indicator identified ex ante, in order to measure the level of achievement of each promoted characteristic/objective pursued;
• With regard to the DNSH principle, the reporting on whether the qualitative/quantitative criteria underlying the assessment of adverse impact indicators have been met.</td></tr>
<tr><td>Inclusion of Sustainability Objectives in Product Governance Processes
• When defining the target market for products, any sustainability-related objectives must be specified.
• Intermediaries are not required to identify a "negative" target market in terms of sustainability objectives.
• Account must be taken of any sustainability-related objectives also during the periodic review of products manufactured and distributed.</td>
<td>Inclusion of ESG Factors in the Decision-Making Process for OICR Management
In order to adopt robust and effective operational mechanisms, managers must pay attention to the following aspects:
• Effective monitoring of compliance with exclusion lists (and their periodic review);
• The definition of investment selection criteria based on disclosures for each ESG pillar, supplemented by a synthetic indicator;
• The verification of compliance with such criteria using up-to-date and reliable information;
• The management of issuers for whom information is wholly or partially unavailable, through pre-defined objective criteria;
• A process that — considered as a whole — is independent, transparent and fair.</td></tr>
</table>

TABLE 11: *Consob Notices.*

lating to the integration of ESG factors into the decision-making processes of collective investment undertakings, and to product-level disclosure transparency. Table 11 provides an overview of the topics addressed in the aforementioned notices. The EU regulatory framework on sustainable finance remains a work in progress, as evidenced by the 2025 review of the Sustainable Finance Disclosure Regulation (SFDR), which represents a significant regulatory intervention aimed at addressing persistent shortcomings in the existing sustainability disclosure framework for financial market participants. Since its initial adoption in 2019, SFDR had been widely criticized for generating legal uncertainty, inconsistent implementation across Member States, and unintended market dynamics, most notably the emergence of de-facto product labels (e.g., Article 8 and Article 9 funds) that were never formally intended as such. The European Commission's proposal acknowledges three principal deficiencies in the original regime, outlined below.

Conceptual Ambiguity Between Entity-Level and Product-Level Disclosures

One of the central criticisms acknowledged in the Commission's amendment proposal concerns the structural ambiguity embedded in the original architecture of SFDR, particularly the distinction between entity-level disclosures (Articles 35 SFDR) and product-level disclosures (Articles 611 SFDR). The original regulation imposed parallel disclosure obligations without sufficiently clarifying the analytical or operational boundaries between them. Therefore, financial market participants frequently encountered interpretative uncertainty regarding:

- Whether sustainability risks should be assessed primarily at the corporate governance level or at the individual financial product level;
- How principal adverse impacts (PAIs) at the entity level should be reconciled with sustainability characteristics advertised at the product level;
- The extent to which entity-level ESG policies were expected to cascade into binding product-specific commitments.

The Commission's proposal explicitly recognizes that this dual-layer disclosure model generated regulatory overlap and semantic confusion, leading to divergent supervisory interpretations across Member States.

Operational Complexity and Disproportionate Compliance Burden

The second deficiency concerns operational and administrative complexity, a recurring theme in both institutional consultations and stakeholder feedback preceding the 2025 review. The Commission acknowledges that the cumulative effect of SFDR, the EU Taxonomy Regulation, and the Delegated Regulatory Technical Standards (RTS) produced a multi-layered compliance ecosystem that proved especially onerous for small and medium-sized financial market participants. The amendment proposal notes several dimensions of complexity:

- **Data Collection Burden**: Firms were required to gather granular ESG data often unavailable in standardized form, particularly from non-EU issuers or private companies.
- **Methodological Uncertainty**: The absence of harmonized calculation methodologies for sustainability indicators led to inconsistent metrics and increased reliance on external ESG data providers.
- **Duplicative Reporting**: Overlaps with PRIIPs KIDs and other disclosure regimes created redundancies that inflated compliance costs without commensurate informational gains for investors.
- **Empirical regulatory assessments**, including studies referenced in EU im-

pact assessments and academic finance literature, suggest that compliance expenditures rose disproportionately relative to firm size. This created a risk of market concentration, where only larger asset managers possessed sufficient resources to comply effectively.

The Commission's recognition of this imbalance reflects a broader EU regulatory principle of proportionality embedded in Article 5 of the Treaty on European Union.

Persistent Greenwashing Risks and Lack of Standardized Sustainability Indicators

The third deficiency identified by the Commission is the continued vulnerability of the regulatory framework to greenwashing practices, despite the extensive disclosure obligations introduced by SFDR. The amendment proposal acknowledges that transparency requirements alone did not guarantee comparability or verifiability of sustainability claims. Three structural drivers of greenwashing risk are emphasized:

- **Indicator Fragmentation**: The original RTS introduced many principal adverse impact indicators, but without universally accepted measurement standards. This resulted in heterogeneous methodologies and selective reporting.
- **Terminological Flexibility**: Terms such as sustainable investment and environmental or social characteristics allowed wide interpretative discretion, enabling strategic marketing practices.
- **Supervisory Divergence**: National competent authorities adopted varying enforcement thresholds, undermining the uniform application of EU law.

The Commission's review reflects this insight by proposing a rationalization and clarification of sustainability indicators, thereby attempting to shift from quantity of disclosure to quality and comparability.

Core Regulatory Changes Proposed in the 2025 SFDR Review

Structural Reconfiguration of the Product Classification Logic

Although the SFDR was originally conceived as a pure disclosure regime, the amendment proposal introduces a de facto structural shift toward a more explicit categorization logic for financial products. Importantly, the Commission does not formally establish legally binding "labels"; however, the proposal acknowledges that Articles 8 and 9 of the current SFDR have already been interpreted by the market as quasi-labels. The reform seeks to re-engineer the architecture of product disclosure in order to:

- Reduce the semantic ambiguity surrounding "promotion of environmental or social characteristics" (Article 8);
- Clarify the threshold for "sustainable investment objectives" (Article 9);
- Avoid the binary and often misleading perception that Article 9 products are inherently "better" or "more sustainable".

Under the proposed changes, the framework moves toward clearer typological distinctions grounded in objective criteria and predefined sustainability strategies. Rather than eliminating the distinction, the Commission proposes to redefine the informational content and evidentiary requirements attached to each category. The intention is to ensure that classification reflects verifiable investment strategy characteristics rather than marketing language.
In this context, the proposal introduces a more structured taxonomy of product types based on the underlying sustainability strategy. Three broad categories are envisaged:

- ESG Basics products represent the most flexible category: at least 70% of the portfolio must integrate sustainability factors through ESG integration practices, combined with minimum safeguards such as the exclusion of controversial weapons, tobacco and companies violating international norms.
- Transition products, designed to support the transformation toward a low-carbon economy. In this case, at least 70% of the portfolio must contribute to measurable transition objectives, for example by financing companies with credible decarbonisation strategies, science-based emission targets or transition plans. The framework also introduces stricter constraints, including limitations on investments in new fossil-fuel exploration activities.
- Sustainable products,the most demanding category, where at least 70% of the portfolio must be aligned with clearly defined environmental or social objectives. These products are subject to the most stringent exclusion criteria, including comprehensive fossil-fuel restrictions broadly aligned with the standards applied to EU Paris-Aligned Benchmarks.

A critical, yet often overlooked, feature of the SFDR 2.0 proposal is the introduction of a 15% EU Taxonomy alignment threshold, designed as a safe harbour mechanism for high-conviction sustainability strategies. While the headline requirement for the Sustainable and Transition categories remains the 70% allocation to qualifying assets, funds that demonstrate at least 15% portfolio alignment with the EU Taxonomy are automatically deemed compliant with the sustainability criteria associated with those categories.
A detailed comparison of the eligibility criteria and disclosure requirements applicable to each category is provided in Tables 12-16.

Revision of Entity-Level vs Product-Level Disclosure Interactions
A second key modification concerns the relationship between entity-level disclosures (Articles 35 SFDR) and product-level disclosures (Articles 611 SFDR). The amendment proposal introduces mechanisms to reduce duplication and clarify vertical coherence between these two layers. The proposed reform aims to establish a clearer hierarchy and cross-referencing system, whereby entity-level disclosures function as a contextual baseline and product-level disclosures focus strictly on investment strategy implementation. This recalibration is intended to preserve transparency while minimizing administrative redundancy and interpretative inconsistency.

Rationalization of Principal Adverse Impact (PAI) Indicators
One of the most technically significant reforms is the streamlining of Principal Adverse Impact indicators, which in the current regime are widely regarded as excessively numerous and methodologically burdensome.
The amendment proposal seeks to:

- Reduce the number of mandatory indicators;
- Distinguish more clearly between core mandatory metrics and optional supplementary metrics;
- Introduce clearer calculation methodologies and definitional guidance.

The Commission's rationale is grounded in proportionality and data availability concerns. Many firms reported difficulty in obtaining reliable ESG data, particularly from non-listed issuers or third-country entities, leading to estimations, proxies, and inconsistent methodologies. The reform thus represents a shift from extensive but heterogeneous disclosure toward more limited but standardized and comparable metrics.

	Art. 7 Transition Category	Art. 8 ESG Basics Category	Art. 9 Sustainable Category
Qualifying Claim	• *Art. 7(1)* — Products claiming to invest in the **transition** of undertakings, economic activities or other assets towards sustainability, or to **contribute to such transition**. **Focused on transition ambition**: investees need not yet be sustainable but must be on a credible path towards it.	• *Art. 8(1)* — Products (other than those under Arts. 7 and 9) claiming to **integrate sustainability factors** in their investment strategy beyond the mere consideration of sustainability risks. **Lowest-ambition category**. No specific sustainability or transition objective required, integration of ESG factors beyond risk management is sufficient.	• *Art. 9(1)* —Products claiming to invest in sustainable undertakings, sustainable economic activities or other sustainable assets, or to **contribute to sustainability.** **Highest-ambition category.** Investees must already be, or positively contribute to being, sustainable.
Minimum Investment Threshold	**Art. 7(1)(a)**: ≥ 70% of investments must meet a **clear and measurable transition objective** related to sustainability factors (environmental or social), consistent with the binding elements of the investment strategy, measured using appropriate sustainability-related indicator(s).	**Art. 8(1)(a)**: ≥ 70% of investments must **integrate sustainability factors** consistent with the binding elements of the investment strategy, measured using appropriate sustainability-related indicator(s).	**Art. 9(1)(a)**: ≥ 70% of investments must meet a **clear and measurable sustainability objective** related to sustainability factors (environmental and social), consistent with the binding elements of the investment strategy, measured using appropriate sustainability-related indicators.
	Art. 7(1) §3-4: **Safe harbour:** • *EU Climate Benchmarks*: Automatically met for products replicating/managed in reference to an EU Climate Transition Benchmark (CTB) or EU Paris-Aligned Benchmark (PAB). • *EU Taxonomy*: Automatically met where ≥ **15%** of investments are taxonomy-aligned.	**Art. 8**: **No EU Benchmark or Taxonomy safe harbour** for Art. 8 (unlike Arts. 7 and 9).	**Art. 9(1) §3-4**: **Safe harbour:** • *EU Paris-Aligned Benchmark (PAB) only:* Automatically met for products replicating/managed in reference to a PAB. The EU CTB is *not* a safe harbour for Art. 9. • *EU Taxonomy*: Also automatically met where ≥ 15% of investments are taxonomy-aligned.
	Art. 7(3)(c)(iii): **Phase-in period** The 70% threshold need not be reached immediately at launch. A phase-in period may be disclosed in pre-contractual documents; the threshold must be attained at its expiry.	**Art. 8(3)(c)(iii)**: **Phase-in period** The 70% threshold need not be reached immediately at launch. A phase-in period may be disclosed in pre-contractual documents; the threshold must be attained at its expiry.	**Art. 7(3)(c)(iii)**: **Phase-in period** The 70% threshold need not be reached immediately at launch. A phase-in period may be disclosed in pre-contractual documents; the threshold must be attained at its expiry.

TABLE 12: *SFDR 2.0 Proposal: Comparative Table of Articles 7, 8 and 9 [Table 1/5].*

	Art. 7 Transition Category	Art. 8 ESG Basics Category	Art. 9 Sustainable Category
Eligible Investment Approaches	**Qualifying investments** • Portfolios replicating/ managed in reference to an EU Climate Transition Benchmark (CTB) or EU Paris-Aligned Benchmark (PAB) *Art. 7(2)(a);* • Taxonomy-aligned economic activities, including transitional activities under Art. 10(2) Taxonomy Reg. *Art. 7(2)(b);* • Undertakings/activities with a credible transition plan for at least one sustainability factor, proportionate to the size of the undertaking *Art. 7(2)(c);* • Undertakings/activities with credible science-based targets (integrity, transparency, accountability) *Art. 7(2)(d);* • Investments with a credible sustainability-related engagement strategy with defined milestones and escalation actions, combined with (a)–(d) or (h) *Art. 7(2)(e);* • Investments pursuant to Art. 9(2) combined with (a)–(e) *Art. 7(2)(f);* • Investments with a credible transition target at portfolio level (e.g. reduction of financed emissions over time) *Art. 7(2)(g);* • Other investments credibly contributing to transition, with proper justification in disclosures (open-ended catch-all) *Art. 7(2)(h).* Where the objective is **climate change mitigation** (*Art. 7(2) §2*)**,** transition plans, science-based targets and engagement strategies must be compatible with the Paris Agreement and the EU Climate Law (Reg. (EU) 2021/1119).	**Qualifying investments** • Investments with an ESG rating (Reg. (EU) 2024/3005) outperforming the average of the investment universe or reference benchmark *Art. 8(2)(a);* • Investments outperforming the average of the investment universe/reference benchmark on a specific sustainability indicator *Art. 8(2)(b);* • Investments favouring undertakings/activities with a proven positive track record on sustainability factors (processes, performance or outcomes) *Art. 8(2)(c);* • A combination of investments under Art. 7(2) or Art. 9(2) with any of (a), (b), (c) above *Art. 8(2)(d);* • Other investments integrating sustainability factors beyond risk, with proper justification in disclosures (open-ended catch-all) *Art. 8(2)(e).* Art. 8 has the **most flexible list,** focused on relative ESG outperformance and factor integration rather than a specific sustainability or transition objective.	**Qualifying investments** • Portfolios replicating/ managed in reference to an EU Paris-Aligned Benchmark (PAB) *Art. 9(2)(a);* • Taxonomy-aligned economic activities as defined in Art. 1(2) Del. Reg. (EU) 2021/2178 *Art. 9(2)(b);* • Instruments issued under Art. 3 Reg. (EU) 2023/2631 (European Green Bonds) *Art. 9(2)(c);* • Investments financing operations benefiting from a Union budgetary guarantee or financial instruments under Union programmes pursuing environmental or social objectives *Art. 9(2)(d);* • Investments in comparable assets to (a)–(d) [Art. 9(2)(e) — further specification in delegated acts] *Art. 9(2)(e).* Art. 9 has the **most restrictive list**: only well-established EU standards and instruments (EU PAB, Taxonomy, EU Green Bonds, Union-backed instruments). The EU CTB, transition plans and science-based targets available under Art. 7 are *not* included here.
Mandatory Exclusions	**Art. 7(1)(b)** **Exclusion Set A: Mandatory baseline exclusions** The following companies must be excluded from the portfolio.	**Art. 8(1)(b)** **Exclusion Set A: Mandatory baseline exclusions (same as Art. 7)** The following companies must be excluded from the portfolio.	**Art. 9(1)(b)** **Exclusion Set A: Mandatory baseline exclusions (broader scope than Arts. 7 and 8)** The following companies must be

TABLE 13: *SFDR 2.0 Proposal: Comparative Table of Articles 7, 8 and 9 [Table 2/5].*

	Art. 7 Transition Category	Art. 8 ESG Basics Category	Art. 9 Sustainable Category
Mandatory Exclusions	*Art. 7(1)(b)* **Exclusion Set A: Mandatory baseline exclusions** The list mirrors the exclusions established for **EU Climate Benchmarks under Art. 12(1)(a)(b)(c)(d) of Del. Reg. (EU) 2020/1818**: • Companies involved in **controversial weapons;** • Companies in **tobacco cultivation and production;** • Companies in **violation of UNGC principles** or OECD MNE Guidelines; • Companies deriving **revenues from hard coal and lignite** above applicable thresholds.	*Art. 8(1)(b)* **Exclusion Set A: Mandatory baseline exclusions (same as Art. 7)** The list mirrors the exclusions established for EU Climate Benchmarks under Art. 12(1)(a)(b)(c)(d) of Del. Reg. (EU) 2020/1818: • Companies involved in **controversial weapons;** • Companies in **tobacco cultivation and production;** • Companies in **violation of UNGC principles** or OECD MNE Guidelines; • Companies deriving **revenues from hard coal and lignite** above applicable thresholds.	*Art. 9(1)(b)* **Exclusion Set A: Mandatory baseline exclusions (broader scope than Arts. 7 and 8)** excluded from the portfolio. The list mirrors the **full scope** of exclusions established for **EU Climate Benchmarks under Art. 12(1) of Del. Reg. (EU) 2020/1818 (all sub-points)**: • Companies involved in **controversial weapons**; • Companies in **tobacco cultivation and production;** • Companies in **violation of UNGC principles** or OECD MNE Guidelines; • Companies deriving **revenues from hard coal and lignite** above applicable thresholds; • Broader fossil fuel revenue thresholds covering **oil and gas** (not required under Arts. 7 and 8).
	Art. 7(1)(c) **Exclusion Set B: New fossil fuel projects** • Companies developing **new exploration / extraction / distribution / refining projects** for hard coal, lignite, oil or gas; • Companies developing new projects for, or without a phase-out plan from, **hard coal or lignite for power generation.**		*Art. 9(1)(c)* **Exclusion Set B: New fossil fuel projects (same as Art. 7)**
	Exception: investments in use-of-proceeds bonds under Art. 3 Reg. (EU) 2023/2631 (EU Green Bonds), or where proceeds do not fund excluded activities, are exempt from Exclusion Set A. *Art. 7(1)(b)(i)(ii)*	**Same exception** for use-of-proceeds instruments as under Art. 7. *Art. 8(1)(b)(i)(ii)*	**Art. 9 applies the most extensive exclusion set**: it references **all of Art. 12(1) Del. Reg. (EU) 2020/1818** (*vs. only (a)–(d) for Arts. 7 and 8*) and **Exclusion Set B**. Same use-of-proceeds exception applies with additional conditions. *Art. 9(1) §5–6*

TABLE 14: *SFDR 2.0 Proposal: Comparative Table of Articles 7, 8 and 9 [Table 3/5].*

	Art. 7 Transition Category	Art. 8 ESG Basics Category	Art. 9 Sustainable Category
Principal Adverse Impacts (PAI) Obligation	**Mandatory PAI disclosure Art. 7(1)(d)** • Financial products must **identify and disclose the principal adverse impacts** of their investments on sustainability factors, and **explain any actions** taken to address those impacts. Financial market participants may comply in full or in part by using appropriate **sustainability-related indicators**. Flexibility on the specific form of PAI disclosure is preserved. *Art. 7(1) §2*	**No mandatory PAI disclosure under Art. 8** • Art. 8 does **not** require identification or disclosure of principal adverse impacts at product level. This is the most significant distinction from Arts. 7 and 9, reflecting the lower sustainability ambition of the ESG basics category. ***Note:*** entity-level PAI disclosures (former Arts. 4–5 SFDR) are entirely deleted by the proposal. Art. 8 does not reinstate them at product level either.	**Mandatory PAI disclosure Art. 9(1)(d)** • Same obligation as Art. 7(1)(d): financial products must **identify and disclose the principal adverse impacts** of their investments on sustainability factors, and **explain any actions** taken to address those impacts. Same flexibility as Art. 7: compliance in full or in part by using appropriate sustainability-related indicators is permitted. *Art. 9(1) §2*
Use of the "Impact" Label	**Available Art. 7(4) and Art. 2(26)** • An Art. 7 product may additionally qualify as a "***sustainability-related financial product with impact***" if it has as its objective the generation of a pre-defined, positive and measurable social or environmental impact. • **Additional required disclosures**: (i) the intended impact(s) with a pre-set impact theory of change; (ii) provisions to measure, manage and report on the desired impact and investor contribution. *Art. 7(4)(a)(b)* Use of the term "impact" in the product name is reserved exclusively for this sub-category. *Art. 13 (naming rules)*	**Not available for Art. 8** • Art. 8 products cannot qualify as sustainability-related financial products with impact. • The impact sub-category is restricted to products categorised under Art. 7 or Art. 9 (Art. 2(26) of the proposal refers exclusively to those two categories).	**Available Art. 9 and Art. 2(26)** • **Same as Art. 7**: an Art. 9 product may qualify as a "sustainability-related financial product with impact" if it pursues a pre-defined, positive and measurable impact with a pre-set theory of change and measurement/management/reporting provisions. *Art. 7(4) applicable by reference.* Use of the term "impact" in the product name is reserved exclusively for this sub-category. *Art. 13 (naming rules)*
Required Pre-Contractual & Periodic Disclosures	**Mandatory disclosures Art. 7(3)** • *Art. 7(3)(a)* Statement that the product meets the Art. 7(1) conditions; • *Art. 7(3)(b)* Description of the transition-related objective(s); • *Art. 7(3)(c)(i)* Description of the transition strategy to comply with the 70% threshold; • *Art. 7(3)(c)(ii)* Choice and relative share of investments under Art. 7(2); • *Art. 7(3)(c)(iii)* Any applicable phase-in period;	**Mandatory disclosures Art. 8(3)** • *Art. 8(3)(a)* Statement that the product meets the Art. 8(1) conditions; • *Art. 8(3)(b)* Description of the sustainability factors integrated; • *Art. 8(3)(c)(i)* Description of the ESG integration strategy to comply with the 70% threshold; • *Art. 8(3)(c)(ii)* Choice and relative share of investments under Art. 8(2); • *Art. 8(3)(c)(iii)* Any applicable phase-in period;	**Mandatory disclosures Art. 9 (mirror of Art. 7(3))** • Statement that the product meets the Art. 9(1) conditions; • Description of the sustainability objective(s); • Description of the sustainability strategy to comply with the 70% threshold; • Choice and relative share of investments under Art. 9(2); • Any applicable phase-in period;

TABLE 15: *SFDR 2.0 Proposal: Comparative Table of Articles 7, 8 and 9 [Table 4/5].*

	ART. 7 TRANSITION CATEGORY	ART. 8 ESG BASICS CATEGORY	ART. 9 SUSTAINABLE CATEGORY
Required Pre-Contractual & Periodic Disclosures	**MANDATORY DISCLOSURES** **Art. 7(3)** • *Art. 7(3)(d)* Where pursuing an environmental objective: statement on whether and to what extent the EU Taxonomy is used to meet the 70% requirement; • *Art. 7(3)(e)* Sustainability-related indicator(s) for measuring compliance and progress; actions to address underperforming assets; • *Art. 7(3)(f)* Statement of compliance with exclusions (Set A and Set B) and any additional exclusions set by the financial market participant; • *Art. 7(3)(g)* Data sources used to inform (b)–(e).	**MANDATORY DISCLOSURES** **Art. 8(3)** • *Art. 8(3)(d)* Sustainability-related indicator(s) for measuring compliance and progress, actions to address underperforming assets; • *Art. 8(3)(e)* Statement of compliance with exclusions (Set A) and any additional exclusions; • *Art. 8(3)(f)* Data sources used to inform (b)–(e). **Key omission vs Arts. 7 and 9** No requirement to disclose on **EU Taxonomy use** (no equivalent of Art. 7(3)(d)) and no **PAI-related disclosure**.	**MANDATORY DISCLOSURES** **Art. 9 (mirror of Art. 7(3))** • Where pursuing an environmental objective: statement on whether and to what extent the EU Taxonomy is used to meet the 70% requirement; • Sustainability-related indicators for measuring compliance and progress; actions to address underperforming assets; • Statement of compliance with exclusions (Sets A and B) and any additional exclusions; • Data sources used; • **Additional**: Impact sub-category Art. 7(4) applicable. Where qualifying as a product with impact: (i) intended impact(s) with pre-set theory of change; (ii) provisions to measure, manage and report on the desired impact, including investor contribution.
Permitted Terms in Names & Marketing	**PERMITTED TERMS** **Art. 13 (as amended by COM(2025) 841)** • Sustainability-related terms permitted in names and marketing; • "**Transition**"-related terms may be used; • "**Impact**" in the product name: permitted only for products qualifying under the Art. 7(4) impact sub-category (Art. 2(26)); • "**Sustainable**" / "**sustainability**": not available for Art. 7 products (reserved for Art. 9).	**PERMITTED TERMS** **Art. 13 (as amended by COM(2025) 841)** • Sustainability-related terms permitted in names and marketing; • **ESG-related terms** may be used in names and marketing; • "**Impact**" and "**sustainable**" / "**sustainability**": not available for Art. 8 products, reserved for Arts. 7 and 9 respectively; • Art. 8 products that invest in categorised products (Art. 9a) may include **sustainability-related claims** in marketing communications but not in their names.	**PERMITTED TERMS** **Art. 13 (as amended by COM(2025) 841)** • Sustainability-related terms permitted in names and marketing; • "**Sustainable**" / "**sustainability**"-related terms may be used; • "**Impact**" in the product name: permitted only for products qualifying under the Art. 9 impact sub-category (Art. 2(26)); • "**Transition**": not available for Art. 9 products (reserved for Art. 7).

TABLE 16: *SFDR 2.0 Proposal: Comparative Table of Articles 7, 8 and 9 [Table 5/5].*

This change directly addresses greenwashing risk by enhancing comparability while simultaneously reducing compliance burdens, especially for smaller asset managers.

Alignment with the PRIIPs Framework
A central institutional reform in the proposal is the integration of sustainability disclosures with the PRIIPs Key Information Document (KID) regime and the repeal of Commission Delegated Regulation (EU) 2022/1288. This measure is designed to enhance regulatory coherence across EU financial disclosure legislation. Under the current system, sustainability information often appears in multiple disclosure formats - prospectuses, periodic reports, websites, and PRIIPs KIDs - leading to fragmentation and investor confusion. The amendment proposal aims to:

- Consolidate sustainability information into standardized investor-facing documents;
- Reduce duplication reporting requirements;
- Improve comparability of sustainability information across financial products.

The repeal of the Delegated Regulation reflects the Commission's recognition that the previous technical standards contributed to complexity and regulatory layering. The new approach favors horizontal integration across disclosure regimes rather than vertical accumulation of technical standards.

Enhanced Terminological Precision and Legal Definitions
Another core change involves the refinement of key legal definitions, including sustainable investment, environmental or social characteristics, and sustainability risk. The amendment proposal seeks to reduce interpretative elasticity by introducing more precise definitional criteria and clearer references to measurable indicators. This definitional tightening is directly connected to the Commission's broader objective of mitigating greenwashing risks. By narrowing the scope of permissible interpretation, the regulation aims to reduce the gap between marketing language and actual investment practice. The emphasis shifts toward verifiability and auditability rather than purely narrative disclosure.

Proportionality and Simplification Measures
Finally, the proposal embeds a stronger principle of proportionality, explicitly acknowledging the uneven compliance capacity across market participants. Simplification measures include:

- Reduced indicator sets for smaller firms;
- Clearer templates and standardized reporting formats;
- Removal of redundant disclosure obligations.

These measures are intended not only to lower administrative burdens but also to prevent unintended market concentration effects whereby compliance complexity advantages large incumbents. Industry feedback, such as that provided by ICMA, generally welcomes these proportionality measures while cautioning against excessive residual complexity.

Empirical Evidence on Regulatory Effectiveness and ICMA feedback

Some empirical assessment of the effectiveness of SFDR-style regulation are provided in the NBER working paper [1]. The study evaluates whether disclosure regulation materially altered fund behavior or investor allocation patterns.

Main Findings
The paper identifies limited real-economy and portfolio effects following the introduction of SFDR:

- Marginal portfolio reallocation: Funds categorized under sustainability provisions showed only modest shifts in asset composition.
- Investor response asymmetry: Retail investors reacted more strongly to sustainability classifications than institutional investors.
- Greenwashing persistence: While disclosure requirements improved transparency, they did not fully eliminate strategic relabeling or superficial ESG integration.

The empirical findings suggest that disclosure regulation alone is insufficient to drive substantive sustainability outcomes. This evidence implicitly supports the Commission's move toward greater standardization and clearer categorization in the 2025 amendment.
Moreover, the International Capital Market Association (ICMA) response provides an industry-oriented perspective on the proposed amendments [6]. ICMA generally welcomes the Commission's intent to simplify and clarify the framework but raises several concerns, specifically:

- **Regulatory coherence**: Alignment with PRIIPs is viewed as a constructive step toward reducing fragmentation.
- **Indicator rationalization**: The reduction of mandatory sustainability metrics is seen as proportionate and pragmatic.
- **Legal clarity**: Greater definitional precision is considered beneficial for compliance predictability.
- **Residual complexity**: ICMA argues that the framework remains overly technical, potentially undermining accessibility for non-specialist investors.
- **Transition risks**: Frequent regulatory adjustments may create uncertainty for long-term product design and capital allocation strategies.
- **International competitiveness**: Concerns are expressed that excessive compliance costs could disadvantage EU financial institutions relative to non-EU competitors.

MIFID II Directive

Within the framework of the MiFID II Directive (2014/65/EU), the integration of environmental, social, and governance (ESG) considerations into the organisational and conduct requirements of firms providing investment advice and portfolio management became applicable in August 2022 with the adoption of Commission Delegated Regulation (EU) 2021/1253, which amended Commission Delegated Regulation (EU) 2017/565. This regulatory intervention was introduced in a context characterised by the proliferation of financial products with varying sustainability features and the emergence of greenwashing risks, resulting in the incorporation of ESG factors into legally binding obligations for investment firms.
A first amendment concerned the introduction of the concept of "sustainability preferences", defined by reference to the categories and disclosure requirements set out in EU Taxonomy and SFDR Regulations, together with the concepts of "sustainability factors" and "sustainability risk". The suitability assessment was consequently linked to definitions already present in European legislation applicable to financial products.
The regulation also required the inclusion of "sustainability risk" in the risk management policies and procedures of investment firms. Firms were required to consider environmental, social, or governance events or conditions that could have a negative, actual or potential, impact on the value of investments, integrating such factors into internal control systems and governance arrangements.
A further amendment concerned the man-

agement of conflicts of interest. Firms were required to assess situations in which the inclusion of sustainability characteristics in products or distribution strategies could create a disadvantage for clients' interests, implementing the measures provided under the MiFID framework for identifying and managing conflicts.
With regard to disclosure obligations, it was established that firms should provide, where relevant, a description of the "sustainability factors" considered in the process of selecting financial instruments. This provision related to how ESG criteria were applied in advisory or portfolio management activities.
Finally, the "sustainability preferences" of a client or potential client were included among the elements of the suitability assessment. In addition to information on investment objectives, financial situation, and risk tolerance, firms were required to record clients' expressed sustainability preferences and verify their alignment with the products or services offered.
In cases where no instruments met these preferences, the regulation provided that the firm could proceed subject to the adaptation of the sustainability preferences decided by the client after receiving adequate information.
Overall, these amendments resulted in the integration of ESG elements into governance processes, risk management, conflict-of-interest controls, disclosure obligations, and suitability assessment procedures under the MiFID framework.

AIFMD Directive

With reference to the regulatory framework governing alternative investment funds, the AIFMD II Directive (2024/927/EU), published in the Official Journal of the European Union on 26 March 2024, does not contain specific articles dedicated to ESG risks. nonetheless, sustainability considerations are explicitly acknowledged in Recital (38) of the directive, which states that "*AIFMs and UCITS management companies should integrate environmental, social, and governance (ESG) parameters into the governance and riskmanagement rules used to support their investment decisions*" and that they should "*apply governance and riskmanagement rules to their investment decisions and to their assessment of relevant risks, including environmental, social, and governance risks*" in support of their investment decisions. In line with this recital, managers of alternative investment funds are expected to integrate ESG parameters into governance and riskmanagement frameworks, ensuring that appropriate resources, expertise, procedures, and monitoring systems are established to identify and manage sustainabilityrelated risks as part of the broader risk management and internalcontrol obligations imposed by Union law.

UCITS Delegated Directive

Within the framework governing UCITS funds, environmental, social, and governance (ESG) risks have been incorporated into the organisational, risk management, and internal control requirements of UCITS management companies through Commission Delegated Directive (EU) 2021/1270 of 21 April 2021, transposed into Italian law by CONSOB Resolution N. 22430 of July 2022. This regulatory intervention addresses the need to integrate sustainability risks into the management of collective investment schemes.
The first amendment introduced the concepts of "sustainability risk" and "sustainability factors", as defined in SFDR Regulation. Management companies are required to identify and assess environmental, social, or governance events or conditions that could have a negative, actual or potential, impact on the value of UCITS portfolios, and to integrate these considerations into their internal control and risk management systems.

Legislation	Scope of Entities	Subject Matter	Entry into Force
MiFID II Directive	**Asset Managers / Inv. Firms** • Investment firms authorised to provide investment services and activities (any size) *Art. 1(1), Art. 4(1)(1).* **Banks** • Credit institutions providing investment services *Art. 1(3)* **Markets / Infrastructures** • Operators of regulated markets *Art. 1(1)* • Data reporting service providers (DRSPs) *Art. 1(1)* **ESG focus:** Investment firms and banks providing investment advice and individual portfolio management → integration of sustainability preferences in suitability assessment under *Art. 25* MiFID II (introduced by Del. Reg. (EU) 2021/1253). No size threshold.	INVESTMENT SERVICES, PRODUCT GOVERNANCE AND SUSTAINABILITY • *Art. 16(3)* Product governance: target market definition with ESG integration (Del. Reg. (EU) 2021/1253); • *Art. 24* General conduct requirements: adequate information, no conflicts of interest; • *Art. 25(2)* Suitability assessment: collection of client sustainability preferences; • *Art. 27* Best execution policy; • *Art. 28* Aggregated order handling. Sustainability preferences must be collected after the risk profile assessment and supplement (without replacing) the suitability evaluation	**Entry into force** *02.07.2014* **Application (after delay)** *03.01.2018* **ESG delegated acts** *(Del. Dir. 2021/1269 and Del. Reg. 2021/1253) 02.08.2022* **Italy** *Legislative Decree 3 August 2017, N. 129 – in force 08.09.2017* ***ESG amendments**: CONSOB Resolution N. 22529, 03.11.2022* *Amendment to the Intermediaries Regulation (Resolution N. 20307/2018)*
AIFMD II Directive	**Asset Managers / EU AIFMs** • EU Alternative Investment Fund Managers managing AIFs (of any type) *Art. 1 (amending Art. 2 AIFMD)* • Managers of EuVECA, EuSEF, ELTIF *Art. 2(3)(4) Dir. 2024/927* **Non-EU AIFMs** • Non-EU Alternative Investment Fund Managers marketing AIFs in the EU *Art. 1 Dir. 2024/927* **Simplified regime** (*Art. 3 AIFMD*, unchanged): AUM <€100M (with leverage) or <€500M (without leverage, lock-up ≥5 years) → simplified registration, not full authorization.	AMENDMENTS TO GOVERNANCE AND AIF MANAGEMENT • *Art. 1 (amending Art. 7-8)* Authorisation: reinforced requirements for new AIFMs; • *Art. 1 (amending Art. 15a, new)* Loan Originating Funds (LOFs): new specific regime; maximum leverage 300% (open-ended) / 175% (closed-ended); • *Art. 1 (amending Art. 16)* Liquidity management for open-ended AIFs: obligation to adopt at least one Liquidity Management Tool (LMT) from Annex V (e.g. gates, swing pricing); • *Art. 1 (amending Art. 20)* Delegation of functions: stricter requirements for notification to national competent authorities (NCAs), enhanced ESMA powers;	**Entry into force** *15.04.2024* **Transposition deadline** *16.04.2026* **Italy** *transposition in progress (March 2026)* *The Ministry of Economy and Finance and the Italian legislature are required to amend Legislative Decree 58/1998 (Consolidated Finance Act) and secondary regulations issued by the Bank of Italy and CONSOB.*

***TABLE 17:** Comparison between MiFID II, AIFMD II and UCITS Delegated Directive [Table 1/2].*

Legislation	Scope of Entities	Subject Matter	Entry into Force
AIFMD II Directive		**Amendments to governance and AIF management** • *Art. 1 (amending Art. 21)* Depositaries: possibility of permanent cross-border appointment; • *Art. 1 (amending Art. 24)* Reporting to authorities: expanded data reporting requirements; • *Art. 2 and 3* Amendments to the UCITS Directive and EuVECA/EuSEF/ELTIF regulations.	
UCITS Delegated Directive	**Asset Managers (UCITS)** • UCITS management companies within the meaning of Dir. 2009/65/EC (UCITS Directive) *Art. 1 Del. Dir. 2021/1270;* • Investment management companies, SICAVs and equivalent entities authorised for collective management of harmonised UCITS. **Exclusions:** Self-managed SICAVs without an external management company. Does not apply to AIFMs (covered by AIFMD). No specific size threshold.	**ESG integration in UCITS management** • *Art. 1 (amending Art. 3)* Organisational requirements: integration of sustainability risks in organisational structures and decision-making processes; • *Art. 1 (amending Art. 12)* Conflicts of interest management: consideration of sustainability risks; • *Art. 1 (amending Art. 18)* Investment process: integration of sustainability risks in pre-investment due diligence and ongoing monitoring; • *Art. 1 (amending Art. 23)* Risk management: sustainability risks are an autonomous risk category within the risk management system. Does not duplicate public disclosure obligations (covered by SFDR); aligns the internal governance framework of UCITS management companies.	**Entry into force** *01.08.2021* **Italy** *Bank of Italy Regulation (23.12.2022)* *Update to the Regulation on Collective Asset Management Integrates sustainability risks into the organisational and risk management framework of investment management companies under Legislative Decree 58/1998 (Consolidated Finance Act)*

TABLE 18: *Comparison between MiFID II, AIFMD II and UCITS Delegated Directive [Table 2/2].*

In accordance with the principle of proportionality, management companies are required to allocate the necessary resources and competences to enable the effective integration of sustainability risks into operational and organisational processes.

This includes establishing procedures, staff expertise, and monitoring mechanisms adequate to ensure compliance with ESG-related obligations.

Finally, the directive requires that ESG risks are integrated into the systems for identifying and managing conflicts of interest. Management companies must assess whether the inclusion of sustainability factors in investment products or management strategies could create a disadvantage for investors and implement the appropriate measures to prevent or mitigate such conflicts.

ESG Regulation in Investment Decision-making: Implication for Investment Intermediaries

Europe remains the world's largest market for sustainable funds, and the proposed revisions under SFDR 2.0 reflect a deliberate ambition to rationalise the existing framework along three interconnected lines (i.e. first, the obligation to produce entity-level disclosures on principal adverse impacts would be abolished, simultaneously removing the redundancies that have long characterised the relationship between SFDR and the CSRD; second, product-level disclosure requirements would be substantially streamlined, with standardised templates replacing the current, more burdensome reporting structure and reducing the number of sustainability indicators that asset managers are required to report; and finally, and perhaps most consequentially from a market-structure perspective, the proposal envisages the creation of three distinct product categories *Sustainable products*, *Transition products*, and *ESG basics*). The anticipated reduction in costs associated with the current SFDR is widely regarded as one of the reform's most tangible benefits, though it would be premature to overlook the substantial transitional burden that asset managers will face in the near term, linked to the reclassification of existing funds, the revision of prospectuses, and the realignment of investment guidelines to the new requirements.

The most recent market data, published by Morningstar [7] for the third quarter of 2025, offers a revealing lens through which to appreciate both the current vitality of the European sustainable fund market and the scale of the challenge that reclassification entails. Article 8 funds the so-called "light green" products attracted net inflows of €28.3 billion during the quarter, a figure representing a 60% increase on the previous period and accounting for 37% of total EU fund flows. Article 9 funds, meanwhile, continued along a trajectory that has become increasingly difficult to ignore: net outflows of approximately €7.1 billion during the quarter point to a persistent and deepening erosion of investor confidence in the most stringently classified products currently available under the framework. Notwithstanding this divergence, the aggregate assets under management of Article 8 and Article 9 funds reached €6.8 trillion, registering a 6% increase equivalent to €400 billion relative to the preceding quarter, a figure that speaks to the overall resilience of the market even as investor preferences shift within it.

What the aggregate figures alone cannot convey, however, is the complexity of what reclassification under SFDR 2.0 will actually require in practice. The exercise is not, as it might superficially appear, a matter of reassigning existing products to new thresholds on the basis of largely unchanged criteria. On the contrary, it demands a substantive reassessment of each product's eligibility for its intended category, grounded in a detailed analysis of sustainability disclosures and ESG metrics produced through the CSRD and EU Taxonomy frameworks. The ongoing monitoring obligations that would accompany initial classification add a further layer of operational complexity that asset managers cannot afford to underestimate. Where this process is not conducted with the requisite rigour, the exposure to reputational risk is considerable and, in a market environment increasingly sensitive to greenwashing concerns, potentially severe. Of particular note in this regard is the prospective fate of the current Article 8 fund universe: under the proposed framework, a significant share of products presently carrying this classification would fail to meet the eligibility criteria for any of the three new sustainability-oriented categories, effectively migrating into the broader pool of non-sustainable products. The practical consequence would be a structural contraction of the Article 8 universe as presently constituted a shift whose implications for product positioning, investor communication, and competitive dynamics within the European asset management industry merit close attention.

Conclusions

In conclusion, the proposed revision of the SFDR represents a significant evolution of the EU sustainable finance framework. While the original SFDR was primarily conceived as a transparency and disclosure regime, the new proposal effectively moves toward a more structured system of product categorisation, aimed at improving comparability between financial products and reducing greenwashing risks.
For the asset management industry, this shift carries several strategic implications. First, the introduction of clearer product categories based on underlying investment strategies is likely to reshape fund design and product positioning, requiring asset managers to align portfolio construction with predefined sustainability criteria and measurable objectives. Second, the strengthened disclosure framework and the greater reliance on verifiable data will increase the importance of robust ESG data governance, internal methodologies and due diligence processes across the investment lifecycle. At the same time, the proposal also aims to simplify and streamline certain reporting requirements, potentially reducing some operational burdens that have emerged under the current regime.
More broadly, SFDR 2.0 may contribute to a gradual maturation of the European sustainable investment market. By clarifying classification rules and linking sustainability claims more closely to underlying investment strategies, the revised framework is expected to enhance investor confidence and foster greater consistency across products marketed as sustainable. For asset managers, the reform therefore represents both a regulatory challenge and a strategic opportunity: firms capable of integrating sustainability considerations credibly and transparently into their investment processes are likely to strengthen their competitive positioning in an increasingly sustainability-driven investment landscape.

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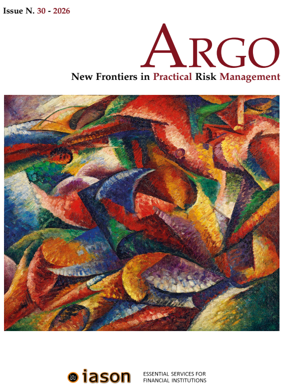

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