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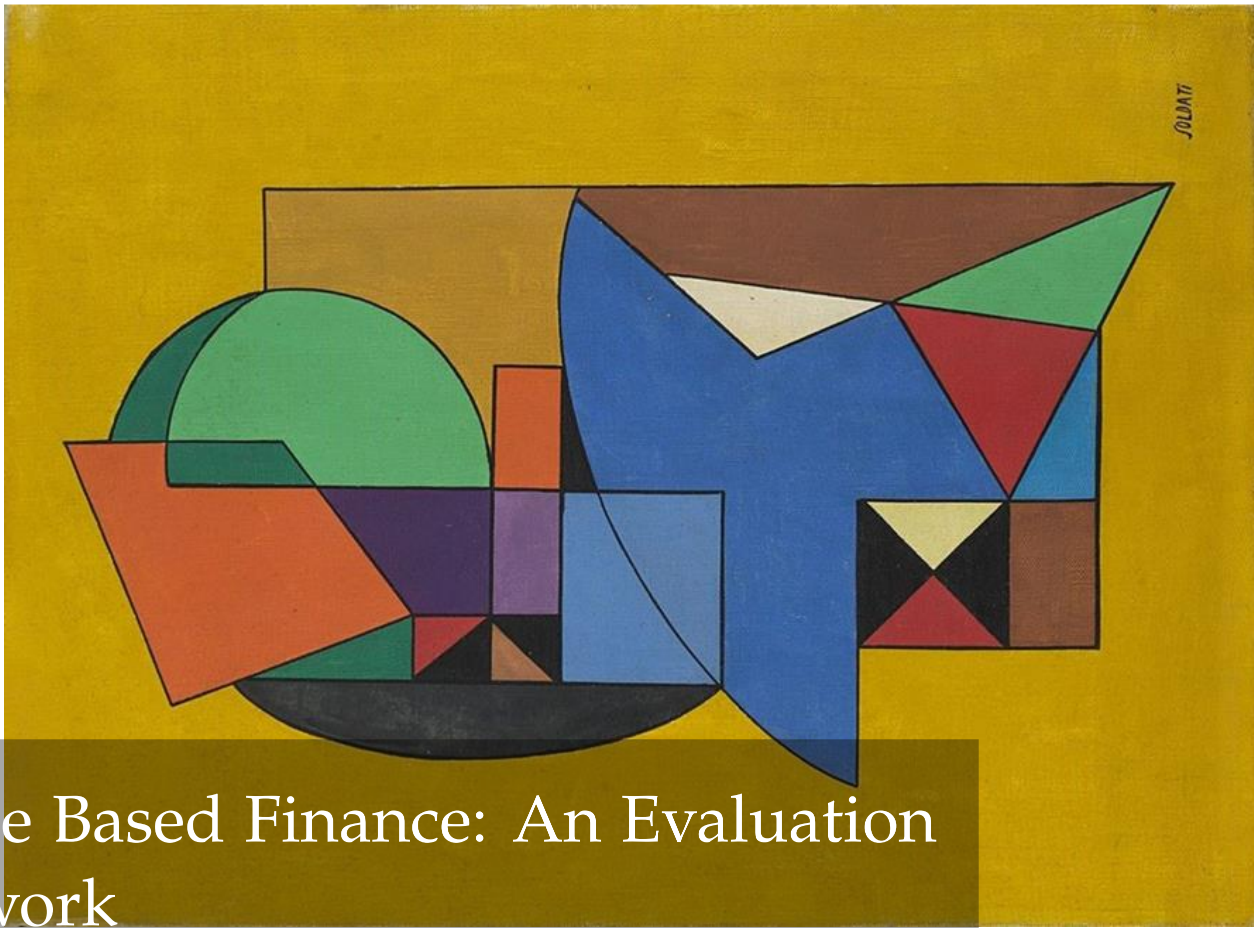

Revenue Based Finance: An Evaluation Framework

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Executive Summary

We present a framework to evaluate Revenue-Based contracts, with different clauses. The framework is used in a practical application.

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Antonio Castagna is currently managing partner and founder of the consulting company iason. He previously was in Banca IMI, Milan, from the 1999 to 2006: there, he first worked as a market maker of cap/floor's and swaptions; then he set up the FX options desk and ran the book of plain vanilla and exotic options on the major currencies, being also responsible for the entire FX volatility trading. He started his carrier in the investment banking in the 1997 in IMI Bank, in Luxemborug, as a financial analyst in the Risk Control Department. He graduated in Finance at LUISS University in Rome in 1995, with a thesis on American options and the numerical procedures for their valuation. He wrote papers on different topics, including credit risk, derivative pricing, collateral management, managing of exotic options risks and volatility smiles. He is also author of the books "FX options and smile risk" and "Measuring and Managing Liquidity Risk", both published by Wiley.

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Revenue-Based Financing (RBF) has recently experienced remarkable growth among private companies. In a nutshell, RBF is an alternative funding model in which investors provide capital to a business in exchange for a fixed percentage of the business's future gross revenues.
There are both advantages and disadvantages for debtor companies when they access this form of financing, and we will examine them below. We will also show that, under the umbrella of RBF, several funding schemes can be grouped together, even though we will focus on the pricing and contract evaluation of arrangements strictly linked to cash flows derived from the company's revenues.
More specifically, we will outline a framework for evaluating RBF contracts that accounts for the main features of typical agreements and the risk factors affecting their value. In some cases, this framework can be easily adapted and extended to different types of agreements, as we will see below.

1. An Overview of Revenue-Based Finance

Revenue-Based Financing is a form of alternative funding that may be preferable to more traditional financing contracts for several reasons. Firstly, it is non-dilutive, as it does not involve equity shares; secondly, it allows for flexible repayments, since payments are proportional to the revenues generated by the business; thirdly, unlike most traditional loans, no collateral is required, whether in the form of personal guarantees or assets.
To these advantages, one can also add the typically faster access to funding offered by RBF compared to loans granted through the banking channel, and the alignment of interests between the debtor company and the financing investor, since the loan's repayment depends on actual future business revenues.
On the other hand, RBF also has some disadvantages: the cost of capital is typically higher than traditional funding; it is not ideal for seasonal or volatile businesses, as investors prefer constant and stable repayments; the impact on available cash can be significant, especially when repayment is based on monthly revenues; finally, the size of funding is limited and usually smaller than equity rounds or traditional loans.
Different funding arrangements can fall under the definition of "Revenue-Based Finance". In Table 1, we provide a quick summary of the types of contracts and their main repayment terms. The first three-Merchant Cash Advances, Royalty-Based Financing, and SaaS (Software-as-a-Service) Financing-are the only ones strictly aligned with the definition provided above: repayment derives from the stream of revenues generated by the business. These revenues can originate, respectively, from sales, from fees related to royalties, or from fees for the use of software products.
The last two types of contracts, Invoice Financing and Inventory Financing, are indirectly related to the revenues of debtor companies. Invoice Financing is currently a very common funding product for working capital, and many web-based platforms provide the infrastructure for smooth processing of all phases of the negotiation between companies and financing investors. The basic features of this financing arrangement are presented and analysed in Castagna [1]. Inventory Financing, although still related to the company's sales, is a form of financing collateralised by the inventory of products: it is suitable only for businesses selling physical goods.
In this work, we will focus on the first three types of contracts. The evaluation framework outlined below can be applied to all of them, their differences being mainly terminological. Apart from the typical (implicit) duration of each of the three contracts, the mechanics of the funding arrangement can be summarized in the following points:

	Merchant Cash Advances	Royalty-based Financing	SaaS Financing	Invoice Factoring	Inventory Financing
Ideal for:	Businesses with steady credit card sales.	Technology, entertainment or product development businesses.	Software companies with predictable subscription revenue.	B2B businesses with unpaid invoices needing immediate cash flow.	Retailers, wholesalers or manufacturers that can use inventory as collateral.
Main contract terms:	Typically up to one year.	Flexible, tied to specific product or project revenue.	3 to 5 years.	70% to 90% of invoice value upfront.	Up to 80% of inventory value.
Repayment terms:	10% to 20% of daily sales.	1% to 10% of revenue from the project or product.	1% to 10% of Monthly Recurring Revenues.	Based on invoice due dates, usually 30 to 120 days.	Typically 6 to 36 months (paid in monthly installments or when inventory is sold).
Cost:	Factor rates of 1.1x to 1.5x times the amount borrowed.	1.5x to 2.5x repayment cap, based on agreed-upon royalties.	Repayment caps typically range from 1.5x to 2.5x.	Fees range from 1% to 5% of invoice value.	May be fees for inventory appraisal, loan origination, prepayment, etc.

TABLE 1: *Types of revenue-based contracts and their main features.*

- The company receives upfront funding from the investor.
- The business agrees to pay a fixed percentage (typically 5-15%) of recurring revenues for a given period; for example, monthly recurring revenues (MRR) or annual recurring revenues (ARR).
- Payments continue until the investor receives a multiple of the original investment (*e.g.*, 1.4x); this amount is usually termed the *repayment cap*.

We can delve into each of these points to analyse some usual contract terms.

Lent Amount

Generally, the debtor company receives a given sum of funds at the inception of the contract; the repayment amount is typically expressed as a multiple of the lent amount.
Sometimes, the contract terms provide for a notional amount from which an upfront fee is deducted. This net amount is what is actually lent to the debtor company, which will then make payments in the future based on business revenues until the notional amount is fully repaid. The difference between the "multiple" and "upfront fee" methods of indicating the lent amount and the repayment cap is merely semantic, with no impact on the financial evaluation when the two alternatives are brought to comparable actual lent amounts.

Expiration Date

As can be easily seen from the points above, there is no fixed end date, and the contract duration depends on revenue performance: higher-than-expected recurring revenues will accelerate repayment and hence shorten the expected duration, while the opposite will occur with lower-than-expected revenues.

In some cases, to protect the lender from an unreasonably long duration if revenues permanently collapse, a term date can be set in the contract. If this is the case, upon the term date all the outstanding lent amount is repaid to the lender, regardless of the percentage that this amount represents with respect to the revenues of the last reference period. When a term date is provided in the contract, the maximum duration of the contract is known, whereas the amounts on each repayment date still depend on business revenues-except for the last one.

Types of Revenues

An RBF contract typically defines "revenues" as the cash receipts generated by the business activity during the reference period: a dedicated bank account that can be monitored by the lender is set up by the borrower, and all cash receipts are paid into it.

A much less common variation, for companies not selling to retail clients but only to other businesses, is the definition of revenues as all invoices issued in a given period, regardless of their actual payment by the client. In this case, outstanding invoices issued during the reference period can be transferred to the lender, who will receive payment directly from the clients. The transfer of invoices can be with or without recourse, depending on whether the borrower remains ultimately liable for any missing payments for any reason.

The cash flows needed for the repayment are those generated by the overall business of the debtor company, but in some cases they can originate from specific receivables that are transferred to the lender. As an example, in royalty-based and SaaS financing, the underlying contracts can be transferred to the lender so that the cash flows paid by the counterparties of these contracts are (possibly fractionally) used to repay the loan. In addition, invoices issued by the debtor company when the loan starts-or to be issued at future dates-can be transferred to the lender, and the cash flows arising when they are paid by the debtor's clients are directly used to repay the debt.

Repayment of the Lent Amount

The repayment of the amount by the debtor is made at the end of each reference period and consists of a share, or rate, of the business revenues or receipts from a royalty or SaaS contract (*e.g.*: 5% of monthly recurring revenues).

Sometimes, a floor can be provided in the contract, so that repayment at the end of a reference period occurs only if business revenues exceed a given amount. If this is the case, a floor may slow down the repayment of the lent sum and extend the duration of the contract, if a term date is not specified.

Default Risk

The main credit risk that the lender bears is that the borrower defaults: if the business of the debtor does not generate sufficient revenues to cover costs, the default of the debtor company becomes inevitable, and the lender will likely suffer a loss on the outstanding lent amount. This case applies to RBF contracts where business cash receipts received by the debtor company are used to repay the debt. When revenues are collected directly by the lender through the assignment of invoices-whose payment produces the cash to repay the debt-we must also consider the default risk of the debtor company's clients, depending on the terms of the RBF contract. In more detail, we may have two cases:

- Full Recourse Loan;
- Limited Recourse Loan.

In the first case, the debtor company is liable for all missing invoice payments by its clients, so the relevant default risk to consider in evaluating an RBF contract is only that of the borrower. In the second case, the lender bears the risk of missed payments, even if no immediate loss on the outstanding capital occurs. In fact, if one or more invoices are not paid, the repayment of the lent amount is smaller than expected, but it can still be completed in the future. Only if the debtor company defaults and business activity stops does the lender suffer a loss on the unpaid portion of the loan.

We stress that the RBF contract provides for the assignment of a fraction of invoices issued in the future, with client companies that may not be identified at the inception of the financing. Thus, we refer to the assignment of invoices to a generic client company: this implies that there is always a surviving generic company whose invoices are transferred to the lender, and the default of this generic client could be material only during a short period of time, *i.e.*, the time span between the assignment and the expiry of the invoices.

Consider, for example, the assignation to the lender of some invoices issued by the debtor company, at some point in the future, so that their payment by the client companies provide the funding for a repayment instalment: in this case, the client company default is relevant only between the assignation date and the expiry date of the invoices. It is quite likely that the RBF contract starts on a date long before the invoices are issued, so that lent amount can be actually seen as a payment in advance for the purchase of invoices to be issued in the future (*e.g.*: one year), that are due according to the terms of the business relationship between the debtor and client companies, (*e.g.*: one month after the issuance).

On the other hand, the default of the debtor company is always relevant, even before the issuance and sale of the account receivables, since when its default occurs, no more revenues will be produced and, consequently, no more invoices can be transferred. Because the invoices are paid by the buyer of the revenue-based claim at the inception of the contract-possibly before the invoices are issued and transferred-the default of the debtor company likely causes a larger loss.

Also in royalty- and SaaS-based contracts, the default risk of the client company should be included in the evaluation of the RBF according to the type of arrangement, which can provide for full or limited recourse to the borrower, as in the case of invoice assignment. With full recourse, the debtor company (*e.g.*, a SaaS company) bears the client's default risk: if the SaaS or royalty-paying client does not pay or terminates the contract, the borrower is obligated to repay the outstanding amount of the loan, just like a normal debt. Thus, SaaS revenues are the source of the cash flows used to repay the debt, but if they stop for any reason, there is no limitation of liability for the debtor company. In evaluating an RBF contract, the default risk that matters is only that of the debtor, since it is ultimately liable for full repayment of the debt.

With limited recourse, the risk of default of the client company falls on the lender, since repayment of the debt is limited to actual receipts from the SaaS contract. So, if the client defaults or the contract revenue stops for any other reason, repayment ends and the lender cannot recover the full lent amount. In this case, the evaluation of an RBF contract should take into account the default risk of the client company and the default risk of the debtor company if this entails termination of the royalty or SaaS contract with the client, as is usually the case.

In royalty- and SaaS-based loans, the counterparty of the underlying contract is typically identified at the start of the agreement, so we are not considering a generic client but a specific company. This means that the default risk of the debtor's client is relevant at any time during the duration of the RBF contract and not just for limited periods, as we saw before in the case of invoice transfers.

	Cash Advance	Cash Advance with Invoice Assignment	SaaS Financing	Royalty-based Financing
Full Recourse	- Default risk of the debtor company.	- Default risk of the debtor company.	- Default risk of the debtor company.	- Default risk of the debtor company.
Limited Recourse	N/A	- Default risk of the debtor company; - Default risk of the counterparty, relevant for the duration of the assigned invoices.	- Default risk of the debtor company.	- Default risk of the debtor company.

TABLE 2: Relevant default risks to consider in the evaluation of different RBF contracts.

A summary of the relevant default risks for the different types of RBF contracts is shown in Table 2.
In what follows, we will focus on financing contracts strictly referring to the business revenues of the debtor company, since it is not particularly difficult to handle royalty- and SaaS-based contracts using standard approaches. In fact, the stream of receipts deriving from royalties and SaaS contracts is usually well defined in the agreements and does not require any special modeling treatment. If these receipts are stochastic, the modeling approach adopted for business revenues can be straightforwardly applied to royalties or SaaS receipts.

2. Modelling the Revenues of the Debtor Company and Other Relevant Risk Factors

To evaluate RBF contracts, we need a framework that considers all relevant risk factors: we must stress that most of these factors cannot be, directly or indirectly, traded on the market. As such, we have to consider real-world stochastic processes and assume an equilibrium model that includes risk premia, allowing the passage to risk-neutral processes and, hence, risk-neutral evaluation.

Credit Risk

The default of the debtor company is modeled following a reduced-form approach, *i.e.*, by directly modeling the default event as a jump process. In more detail, let $\Lambda_t = \mathbf{1}_{t\geq\tau}$ be the indicator function of the event "default" at time τ. It is a stochastic process, specifically a Poisson process. If λ is assumed to be the instantaneous arrival intensity rate, then we have:

$$\mathbf{E}[d\Lambda_t] = \lambda\, dt.$$

We assume that the default intensity follows (in the real world) as square-root mean reverting process, or a CIR process (from the Cox, Ingersoll and Ross, who used this process for the instantaneous interest rate, see [3]):

$$d\lambda(t) = \kappa_\lambda[\theta_\lambda - \lambda(t)]dt + \sigma_\lambda\sqrt{\lambda(t)}\, dZ_\lambda(t). \tag{1}$$

We define the survival probability $\mathbf{SP}(t,T)$ between time t and T as:

$$\mathbf{SP}(t,T) = \mathbf{E}\left[e^{-\int_t^T \lambda(s)ds}\right], \tag{2}$$

which admits an explicit solution in the CIR setting, as shown below.
For the evaluation of some revenue-based contracts, it is useful to introduce an additional risk factor: the probability of default of the clients of the debtor company. The receivables transferred to the lender generate the cash flows that repay the debt: when the debtor's client company goes bust, the revenue-based claim suffers a missed payout (which can be covered by the debtor if full recourse is allowed, see above).
To model this risk, in case the RBF contract provides for the assignment of invoices, we assume that the receivables are paid by a representative generic client company; when the client company can be unequivocally identified, the default refers to this specific company. Let $\Xi_t = \mathbf{1}_{t \geq \tau^c}$ be the indicator function of the event "default" of the client company at time τ^c. As in the case of the debtor company's default, it is a stochastic process, specifically a Poisson process, with instantaneous arrival intensity rate ξ, so that:

$$\mathbf{E}[d\Xi_t] = \xi\, dt.$$

Also in this case, the default intensity follows the square-root mean reverting process:

$$d\xi(t) = \kappa_\xi[\theta_\xi - \xi(t)]dt + \sigma_\xi \sqrt{\xi(t)}\, dZ_\xi(t). \tag{3}$$

The survival probability $\mathbf{SP}^c(t,T)$ between time t and T is:

$$\mathbf{SP}^c(t,T) = \mathbf{E}\left[e^{-\int_t^T \xi(s)ds}\right]. \tag{4}$$

Whether to include or not, and to what extent, the default risk of the client of the debtor company depends on the terms of the RBF contract, as we showed above.

Revenues
Business revenues are modelled with respect to the reference period provided for in the contract: we saw above that the repayment of the loan is made by means of a fraction of the recurring revenues generated in a given period (say, one month). So, we need to model the recurring revenues for this period of time. Let us denote them with V^* and assume they follow the process:

$$dV^*(t) = \mu V^*(t)\, dt + \sigma_V V^*(t)\, dZ_V(t). \tag{5}$$

This is a standard geometric Brownian motion widely used in finance: it is a continuous process, but in practice observed only at the end of each reference period. More specifically, given the duration of the loan contract $[0,T]$, which includes N end-of-period dates $T_1, T_2, \ldots, T_N$, the process $V^*(t)$ will be observed-and hence its value measured-on these N end-of-period dates (for example, at every month-end). It should be stressed that V^* does not represent cumulative revenues over the interval $[0,T]$: it refers only to the revenues for each reference period within the duration of the loan contract, which are fictitiously assumed to follow a continuous process, even though their value matters only at the levels attained on the N end-of-period dates.
Now define two processes: the revenue process $V^*(t)$, which ignores any effect of a default event; the process $V(t)$, which equals $V^*(t)$ before default occurs, that is $V(t) = V^*(t) \cdot \mathbf{1}_{t<\tau}$, and it drops to 0 after the default event, so that it can be written as:

$$V(t) = 0, \;\; t > \tau.$$

Applying the Ito's lemma and the analogous lemma for the Poisson process, one gets the following dynamics for the process $V(t)$:

$$dV(t) = \mu V(t)\, dt + \sigma_V V(t)\, dZ_V(t) + [V(\tau) - V(t)]\, d\Lambda(t). \tag{6}$$

It is useful to note that, since $V(\tau) = 0$, we have that the expectation of the last term of the SDE (6) is equal to $\mathbf{E}[-V(t)d\Lambda(t)]$ and it can be written as:

$$\mathbf{E}[-V(t)d\Lambda(t)] = \mathbf{E}[-V(t)|d\Lambda(t) = 1]\mathbf{E}[d\Lambda(t)] = -V(t)\lambda(t)\, dt,$$

so that the dynamics of the revenues is:

$$dV(t) = [\mu - \lambda(t)]V(t)\, dt + \sigma_V V(t)\, dZ_V(t). \tag{7}$$

The process $V(t)$ may be useful to include in a compact fashion the debtor's default risk in the evaluation formulae.

Interest Rate
Interest rates are modelled through the evolution of the instantaneous rate, which assumed to follow a mean-reverting square root model as in Cox, Ingersoll and Ross [3]:

$$dr(t) = \kappa_r[\theta_r - r(t)]\, dt + \sigma_r \sqrt{r(t)}\, dZ_r. \tag{8}$$

The price at time t of a zero-coupon bond $P(t, T)$ expiring at time T can be calculated as:

$$P(t, T) = \mathbf{E}\left[e^{-\int_t^T r(s)ds}\right], \tag{9}$$

which admits a closed-form solution in our setting (see below). The price of a zero-coupon bond can also be seen as the discount factor applied to future cash flows to compute their present value.

3. Evaluation of Revenue-Based Claims

To evaluate revenue-based claims (or, more generally, contracts), we must consider that most risk factors cannot be hedged with market instruments: no contract on revenues, nor CDS on a small company (the typical debtor), is actively traded on the market; only interest rates can be hedged through contracts traded on the market. Therefore, we cannot resort to pricing via a replicating portfolio *á la* Black&Scholes; instead, we need an equilibrium model such as the one described in Cox, Ingersoll, and Ross [2].

3.1 The PDE of Revenue-Based Claims

A claim on the revenues of a company can be mathematically defined as a function $J(V(t), r(t), \lambda(t), \xi(t), t)$ with a dynamics defined as:

$$\begin{aligned} dJ(V(t), r(t), \lambda(t), \xi(t), t) &= \mu_J(V(t), r(t), \lambda(t), t)\, dt \\ &+ \sigma_J(V(t), r(t), \lambda(t), t) dZ_J + [J(\tau^c) - J(t)]\, d\Xi(t), \end{aligned} \tag{10}$$

The value of the claim depends on time, revenues, interest rates and the default risks of the debtor and client companies. The default of the client company directly affects the value

of the claim through the term $[J(\tau^c) - J(t)], d, \Xi(t)$, whereas the default risk of the debtor company directly affects the revenues and thus indirectly the value of the claim.
By setting $J(\tau^c) = 0$ (*i.e.*: the claim is worth nil when the client company defaults), from Ito's lemma, simplifying the notation, the drift of the claim is:

$$\begin{aligned}
&\frac{1}{2}\sigma_V^2 V^2 J_{VV} + \frac{1}{2}\sigma_r^2 r J_{rr} + \frac{1}{2}\sigma_\lambda^2 \lambda J_{\lambda\lambda} + \frac{1}{2}\sigma_\xi^2 \xi J_{\xi\xi} \\
&+ \mu V J_V + \kappa_r(\theta_r - r) J_r \\
&+ \kappa_\lambda(\theta_\lambda - \lambda) J_\lambda + \kappa_\xi(\theta_\xi - r) J_\xi \\
&- \lambda V J_V - \xi J + J_t - \xi J
\end{aligned} \tag{11}$$

where J_x and J_{xx} indicate, respectively, the first and second derivatives of the claim value function J with respect to the variable x (we have also slightly lightened the notation).
On the other hand, in an economy described in Cox, Ingersoll and Ross [2] and under the assumption in Cox, Ingersoll and Ross [3], no-arbitrage conditions imply that any claim whose value depends on the price of the revenues' value must have an instantaneous drift equal to:

$$\left(r + \pi_V V \frac{J_V}{J} + \pi_r r \frac{J_r}{J} + \pi_\lambda \lambda \frac{J_\lambda}{J} + \pi_\xi \xi \frac{J_\xi}{J} \right) J,$$

where $\pi_i,\ i \in \{V, r, \lambda, \xi\}$ are the market prices of risk for the three factors respectively.
By equating the drift in SDE (10), explicitly derived by means the Ito's lemma, to the drift obtained on no-arbitrage conditions, we have:

$$\begin{aligned}
&\frac{1}{2}\sigma_V^2 V^2 J_{VV} + \frac{1}{2}\sigma_r^2 r J_{rr} + \frac{1}{2}\sigma_\lambda^2 \lambda J_{\lambda\lambda} + \frac{1}{2}\sigma_\xi^2 \xi J_{\xi\xi} \\
&+ (\mu - \pi_V) V J_V + [\kappa_r(\theta_r - r) - \pi_r r] J_r \\
&+ [\kappa_\lambda(\theta_\lambda - \lambda) - \pi_\lambda \lambda] J_\lambda + [\kappa_\xi(\theta_\xi - r) - \pi_\xi \xi] J_\xi \\
&+ J_t - \lambda V J_V - \xi J = rJ.
\end{aligned} \tag{12}$$

In PDE (12), the drift of all risk factors (V, r, λ, ξ) can now be considered risk-adjusted by the appropriate market price of risk, so that the drift of the claim (LHS of the PDE) equates to the yield of a risk-free asset, *i.e.*, the interest rate. The value of any claim depending on the four risk factors is the solution of PDE (12), given the terminal condition and the boundary conditions provided by the pay-out at expiry and, possibly, other contract terms.

3.2 Revenue-Based Zero-Coupon Bond

We will price a revenue-based zero-coupon bond for the different variations of RBF contracts examined above with respect to credit risk. We start with the Full Recourse clause, and then analyse the contracts with the Limited Recourse clause. In both cases, we will consider whether the invoice assignment determines how the repayment is made.

3.2.1 Full Recourse Clause

Base Case

We define the revenue-based zero-coupon bond as an asset paying a fraction ω of the revenues $V(T)$ collected during the reference period ending at time T. Let t be the evaluation time, and let $H(V, r, \lambda, t, T)$ denote the value at time t of the revenue-based zero-coupon bond: it is the solution of PDE (12) with a terminal condition equal to the pay-out. The solution can be expressed as an expectation under the risk-neutral measure Q:

$$H(V,r,\lambda,t,T) = \mathbf{E}^Q\left[e^{-\int_t^T r(s)ds}\omega V(T)\mathbf{1}_{\{\tau>T\}}\right].$$

The indicator function $\mathbf{1}_{\{\tau>T\}}$ accounts for the default of the debtor company, so that the revenue-based zero-coupon bond pays out the fraction of the revenue only if the company survives up to time T.
The expected value $H(V,r,\lambda,t,T)$ is:[1]

$$H(V,r,\lambda,t,T) = H^{FR}(t,T) = \omega P(r,t,T)V_t N(\lambda,t,T), \tag{13}$$

where N(t,T) is:

$$N(\lambda,t,T) = C(t,T)e^{-\lambda(t)D(t,T)}e^{(\mu-\pi_V)(T-t)},$$

$$C(t,T) = \left[\frac{2\phi e^{\frac{(\kappa_\lambda+\phi+\pi_\lambda)(T-t)}{2}}}{(\kappa_\lambda+\phi+\pi_\lambda)\left(e^{\phi(T-t)}-1\right)+2\phi}\right]^{\frac{2\kappa_\lambda\theta_\lambda}{\sigma_\lambda^2}},$$

$$D(t,T) = \frac{2\left(e^{\phi(T-t)}-1\right)}{(\kappa_\lambda+\phi+\pi_\lambda)\left(e^{\phi(T-t)}-1\right)+2\phi},$$

$$\phi = \sqrt{(\kappa_\lambda+\pi_\lambda)^2+2\sigma_\lambda^2}.$$

The quantity $N(\lambda,t,T)$ includes the survival probability of the debtor company, so that we can also write:

$$N(\lambda,t,T) = \mathbf{SP}(t,T)e^{(\mu-\pi_V)(T-t)}.$$

Where $\mathbf{SP}(t,T) = C(t,T)e^{-\lambda(t)D(t,T)}$.
The quantity $P(r,t,T)$ is the price of an interest rate zero-coupon bond:

$$P(r,t,T) = A(t,T)e^{-r(t)B(t,T)},$$

$$A(t,T) = \left[\frac{2\gamma e^{\frac{(\kappa_r+\gamma+\pi_r)(T-t)}{2}}}{(\kappa_r+\gamma+\pi_r)\left(e^{\gamma(T-t)}-1\right)+2\gamma}\right]^{\frac{2\kappa_r\theta_r}{\sigma_r^2}},$$

$$B(t,T) = \frac{2\left(e^{\gamma(T-t)}-1\right)}{(\kappa_r+\gamma+\pi_r)\left(e^{\gamma(T-t)}-1\right)+2\gamma},$$

$$\gamma = \sqrt{(\kappa+\pi_r)^2+2\sigma_r^2}.$$

The proof is in the Appendix A.1.

Assignment of Invoices
We define the revenue-based zero-coupon bond as an asset paying a fraction ω of the revenues at time $V(T)$. Let t be the evaluation time: the pay-out is given by the payment of one or more invoices by the representative client company; the invoices are issued before T with expiry at time $S \geq T$ (T is the end of the reference period of the RBF contract) and are transferred at T to the buyer of the zero-coupon bond.

[1]The solution for the expected value can be derived using standard techniques for affine processes and properties of the square-root mean-reverting SDE.

If the debtor company has not defaulted before T, the transfer of the invoices is possible; otherwise, the revenue-based zero-coupon bond expires worthless. Additionally, with the Full Recourse clause in force, if the client of the company (*i.e.*, the payer of the invoices) defaults, the lender may request full payment from the debtor company, so that its survival up to time S is what really matters in the evaluation of the revenue-based zero-coupon bond, without any need to account for the client's default risk.
Let us denote the value at time t of this revenue-based zero-coupon bond by $H(V,r,\lambda,t,T,S)$: it is the solution of PDE (12) with terminal condition equal to the pay-out. The solution can be expressed as an expectation under the risk-neutral measure Q:

$$H(V,r,\lambda,t,T,S) = \mathbf{E}^Q\left[e^{-\int_t^S r(s)ds}\omega V(T)\mathbf{1}_{\{\tau>S\}}\right].$$

The expected value $H(V,r,\lambda,\xi,t,T,S)$ is

$$H(V,r,\lambda,\xi,t,T,S) = H^{FRA}(t,T,S) = \omega P(r,t,S)V_t\mathbf{SP}(t,S)e^{(\mu-\pi_V)(T-t)}, \qquad (14)$$

where $P(r,t,S)$ and $\mathbf{SP}(t,S)$ are the same functions provided above for the Base case.
The proof in the Appendix A.1.

3.2.2 Limited Recourse Clause

When the RBF contract provides only for Limited Recourse, we need to consider only the case when the repayment is made by assigning to the lender the invoices issued by the debtor company during the reference period. This is addressed in what follows.

Assignment of Invoices
As before, we define the revenue-based zero-coupon bond as an asset paying a fraction ω of the revenues at time $V(T)$: the pay-out is given by the payment of one or more invoices by the representative client company; the invoices are issued before T with expiry at time $S \geq T$ and they are transferred in T to the lender.
Also in this case, only if the debtor company did not default before T the transfer of the invoices is possible, but since no recourse is allowed if the client defaults, only its survival between time T, when the invoices are assigned, and time S, when they are paid. So, the debtor's default is relevant only if it happens between the evaluation time t and the assignment time T, whereas the client's default is relevant between T and S.
The solution of the PDE (12) is expressed as an expectation under the risk-neutral measure Q:

$$H(V,r,\lambda,t,T,S) = \mathbf{E}^Q\left[e^{-\int_t^S r(s)ds}\omega V(T)\mathbf{1}_{\{\tau^c>S|\tau^c>T\}}\right].$$

The expected value $H(V,r,\lambda,\xi,t,T,S)$ is

$$\begin{aligned} H(V,r,\lambda,\xi,t,T,S) &= H^{LRA}(t,T,S) \\ &= \omega V_t P(r,t,S)N(\lambda,t,T)M(\xi,t,T,S). \end{aligned} \qquad (15)$$

The quantity $P(r,t,S)$ and $N(\lambda,t,T)$ are the functions provided above, whereas the quantity $M(\xi,t,T,S)$ is the survival probability of the client company in the period $[T,S]$:

$$M(\xi,t,T,S) = \mathbf{SP}^c(T,S) = E(T,S)e^{-\xi(t)\frac{\eta F(T,S)e^{-\kappa_\xi(T-t)}}{\eta+F(T,S)}},$$

$$E(T,S) = \left[\frac{2\psi e^{\frac{(\kappa_\xi+\psi+\pi_\xi)(S-T)}{2}}}{(\kappa_\xi+\psi+\pi_\xi)\left(e^{\psi(S-T)}-1\right)+2\psi} \right]^{\frac{2\kappa_\xi\theta_\xi}{\sigma_\xi^2}} \left(\frac{\eta}{\eta+F(T,S)} \right)^{\frac{2\kappa_\xi\theta_\xi}{\sigma_\xi^2}},$$

$$F(T,S) = \frac{2\left(e^{\psi(S-T)}-1\right)}{(\kappa_\xi+\psi+\pi_\xi)\left(e^{\psi(T-S)}-1\right)+2\psi},$$

$$\eta = \frac{2(\kappa_\xi+\pi_\xi)}{\sigma_\xi^2(1-e^{-(\kappa_\xi+\pi_\xi)(S-t)})}, \qquad \psi = \sqrt{(\kappa_\xi+\pi_\xi)^2+2\sigma_\xi^2}.$$

When the client is a specific company and not a generic client company, $M(\xi,t,T,S)$ modifies as:

$$M(\xi,t,T,S) = \frac{\mathbf{SP}^c(t,S)}{\mathbf{SP}^c(t,T)}.$$

The proof is in the Appendix A.1.

3.3 Revenue-Based Bond

A revenue-based bond is a claim depending on the future stream of revenues of the debtor company. The stream may vary depending on the terms of the contract that we analysed earlier.
The amount R_{t_0}, defined at time t_0 (which is also the evaluation time), must be reimbursed in variable and stochastic instalments equal to a fraction ω of the future revenues at times, until the amount is fully repaid. At any future date t_i, $i \in \{1,\ldots,n^*,N\}$, R_{t_i} is the outstanding balance to be repaid, with $R_{t_{n^*}} = 0$ (*i.e.*, the outstanding balance is fully repaid by the fraction of the revenues). If a term date t_N is set, then the outstanding amount is fully repaid on that date, so also in this case $R_{t_N} = 0$. Clearly, it is possible that the debt is repaid earlier than the term date, *i.e.*: $t_{n^*} \leq t_N$. The repayment may or may not be made by assigning invoices to the buyer of the bond (the lender), and the contract may provide for the Full or Limited Recourse clause.
It should be noted that the last date t_{n^*} cannot be known with certainty if a term date is not set in the contract, since the full repayment of the debt depends on the future level of revenues.
Finally, we will also consider the possibility that the contract provides for a revenue floor, so that on each date t_i, repayment occurs only if the revenues of the reference period are greater than a given amount Z, *i.e.*, $V(t_i) > Z$.
As we did for the revenue-based zero-coupon bond, we will present evaluation formulae for the different RBF contract terms.

3.3.1 Full Recourse Clause

Base Case
The stream of cash flows of a revenue-based bond is the sum of the fraction ω of future revenues referring to the reference periods ending on dates $\{t_1, t_2, \ldots, t_{n^*}, t_N\}$; each payment reduces the outstanding debt. At any time t_i, the payment is simply the outstanding debt if this is smaller than the fraction of revenues that should be paid. On the other hand, if t_i is also the term date t_N for the repayment, the cash flow is again the outstanding debt, regardless of how much the fraction of revenues equals at that time. Additionally, on each

repayment date t_i, revenues must be above the floor Z. Formally, considering all this, the cash flow K_{t_i} at time t_i can be written:

$$\begin{aligned} K_{t_i} =& \min\left[\omega V^*(t_i), R_{t_{i-1}}\right]\mathbf{1}_{\{V(t_i)>Z\}} \\ &+ \left[R_{t_{i-1}} - \min\left[\omega V^*(t_i), R_{t_{i-1}}\right]\right]\mathbf{1}_{\{t_i=t_N\}} \\ &= \left[\omega V^*(t_i) - \max\left[\omega V^*(t_i) - R_{t_{i-1}}, 0\right]\right]\mathbf{1}_{\{V(t_i)>Z\}} \\ &+ \left[R_{t_{i-1}} - \left[\omega V^*(t_i)\right.\right. \\ &+ \left.\left.\max\left[\omega V^*(t_i) - R_{t_{i-1}}, 0\right]\right]\mathbf{1}_{\{V(t_i)>Z\}}\right]\mathbf{1}_{\{t_i=t_N\}}, \end{aligned}$$

where $R_{t_{i-1}}$ is the amount of the outstanding debt at time t_{i-1}. By setting R_{t_0} equal to the initial amount lent to the debtor company, the outstanding debt at any time t_i can be recursively calculated as:

$$R_{t_i} = R_{t_{i-1}} - K_{t_i}.$$

It should be noted that the quantity K_{t_i}, and hence R_{t_i}, are stochastic, since they depend on the volume of revenues generated by the business activity of the debtor. We are not considering any default risk for the moment, as we want to ascertain how long it takes to repay the debt given the revenues generated in the future. The expected payment K_{t_i} is:

$$\begin{aligned} \mathbf{E}^Q[K_{t_i}] =& \mathbf{E}^Q\left[\min\left[\omega V^*(t_i), R_{t_i}\right]\mathbf{1}_{\{V(t_i)>Z\}}\right. \\ &+ \left.\left[R_{t_{i-1}} - \min\left[\omega V^*(t_i), R_{t_{i-1}}\right]\mathbf{1}_{\{V(t_i)>Z\}}\right]\mathbf{1}_{\{t_i=t_N\}}\right] \\ &= \mathbf{E}^Q\left[\left[\omega V^*(t_i) - \max\left[\omega V^*(t_i) - R_{t_{i-1}}, 0\right]\right]\mathbf{1}_{\{V(t_i)>Z\}}\right. \\ &+ \left[R_{t_{i-1}} - \left[\omega V^*(t_i) + \right.\right. \\ &\left.\left.\left.\max\left[\omega V^*(t_i) - R_{t_{i-1}}, 0\right]\right]\mathbf{1}_{\{V(t_i)>Z\}}\right]\mathbf{1}_{\{t_i=t_N\}}\right], \end{aligned} \tag{16}$$

which can be explicitly computed as:

$$\begin{aligned} \mathbf{E}^Q[K_{t_i}] =& \left[\omega V(t_0)e^{(\mu-\pi_V)(t_i-t_0)} - \mathcal{O}(V^*(t_i), R_{t_{i-1}}, t_0, t_i)\right]\mathbf{PF}(t_0, t_i) \\ &- \left[\mathbf{E}^Q[R_{t_{i-1}}] + \left[\omega V(t_0)e^{(\mu-\pi_V)(t_i-t_0)}\right.\right. \\ &\left.\left.- \mathcal{O}(V^*(t_i), R_{t_{i-1}}, t_0, t_i)\right]\mathbf{PF}(t_0, t_i)\right]\mathbf{1}_{\{t_i=t_N\}}, \end{aligned} \tag{17}$$

where

$$\mathcal{O}(V^*(t_i), R_{t_{i-1}}, t_0, t_i) = \mathbf{E}^Q\left[\max\left[\omega V^*(t_i) - R_{t_{i-1}}, 0\right]\right]. \tag{18}$$

The solution to (18) is based on the Black&Scholes pricing formula for a call option, and it is:

$$\mathcal{O}(V^*(t_i), R_{t_{i-1}}, t_0, t) = \left[\omega V^*(t_0)e^{[\mu-\pi_V](t_i-t_0)}N(d_1) - R_{t_{i-1}}N(d_2)\right], \tag{19}$$

with $N(x)$ is the normal distribution function in x and

$$d_1 = \frac{\ln \frac{\omega V^*(t_0)}{R_{t_{i-1}}} + [\mu - \pi_V + \frac{1}{2}\sigma_v^2](t_i - t_0)}{\sigma_v \sqrt{t_i - t_0}}, \quad d_2 = d_1 - \sigma_v \sqrt{t_i - t_0}.$$

The quantity $\mathbf{PF}(t_0, t_i)$ is the (risk-neutral) probability calculated at t_0 that the revenues at the end of the reference period t_i are higher than the floor Z. The explicit formula is:

$$\mathbf{PF}(t_0, t_i) = \mathbf{E}^Q \left[V(t_i) > Z \right] = N(d_3), \tag{20}$$

with

$$d_3 = \frac{\ln \frac{V^*(t_0)}{Z} + [\mu - \pi_V - \frac{1}{2}\sigma_v^2](t_i - t_0)}{\sigma_v \sqrt{t_i - t_0}}.$$

The expected outstanding amount R_{t_i} is then simply:

$$\mathbf{E}^Q[R_{t_i}] = \mathbf{E}^Q[R_{t_{i-1}} - K_{t_i}], \tag{22}$$

recursively computed. For the valuation of the bond below, the date t_{n^*}-the date when the outstanding debt becomes nil and is fully repaid-is calculated by means of equations (17) and (22). These are used for all cases of RBF contracts we are analysing.
The expected present value at time t_0 of the stream of cash flows is the value of a revenue-based bond, defined as:

$$B^{FR}(R_{t_0}, t_0, t_1, t_N) = \mathbf{E}^Q \left[\sum_{i=1}^{N} e^{-\int_{t_0}^{t_i} r(s) ds} K_{t_i} \mathbf{1}_{\{\tau > t_i\}} \right], \tag{23}$$

where t_1 is the first date when repayment starts and t_N is the term date (if no term date is set, then $t_N = \infty$, so that the summation ends at t_{n^*}, or the earliest time when the outstanding balance of the debt is fully repaid, $R_{t_{n^*}} = 0$). It is worth noting that the debt may be fully repaid before the term date t_N, so that some addends of the summation will be nil. The value $B^{FR}(R_{t_0}, t_0, t_1, t_N)$ should correspond to the amount actually lent to the debtor company at time t_0, given the contract terms. This bond does not pay any coupon, but only stochastic instalments, depending on the revenues, that are used to repay R_{t_0}. The interest rate and the compensation for credit risk are implicit in the difference between the lent amount $B(R_{t_0}, t_0, t_1, s)$ and the amount R_{t_0} that must be repaid.
By calculating the expectations, we get the explicit formula:

$$\begin{aligned} B^{FR}(R_{t_0}, t_0, t_1, t_N) = \sum_{i=1}^{N} \Bigg[& [H^{FR}(t_0, t_i) \\ & - P(r, t_0, t_i) \mathcal{O}(V^*(t_i), R_{t_{i-1}}, t_0, t_i)] \mathbf{PF}(t_0, t_i) \mathbf{SP}(t_0, t_i) \Bigg] \\ & + P(r, t_0, t_N) \Bigg[\mathbf{E}^Q[R_{t_{N-1}}] - [H^{FR}(t_0, t_{N-1}) \\ & - \mathcal{O}(V^*(t_N), R_{t_{N-1}}, t_0, t_N)] \mathbf{PF}(t_0, t_i) \Bigg] \mathbf{SP}(t_0, t_N). \end{aligned} \tag{24}$$

Assignment of Invoices

The stream of cash flows of a revenue-based bond is the same as above: in this case, t_{n^*} is the earliest between the time when the outstanding balance of the debt is fully repaid, $R_{t_{n^*}} = 0$, and the term date t_N, when the outstanding balance must be paid in full anyway. Compared with the base case, the only difference lies in the occurrence of the cash flows, each one at a period s after the relevant end date of the reference period (for simplicity's sake, s is constant but it can be made also variable without any substantial change to the formulae below). The invoices are still related to the revenues generated in each reference period.

Equations (17) and (22) are used to calculate the expected repayments and outstanding debt at each period, and the date t_{n^*} when the debt is expected to be fully repaid. The valuation formula should be modified in two ways: first, the discount of the expected cash flows is between t_0 (the evaluation date) and the payment dates $\{t_1 + s, t_2 + s, \dots, t_{n^*} + s, t_N + s\}$; second, the debtor's survival probability is also calculated up to the payment dates, since it is liable for the payment in case the client misses the payment. All the remaining terms are the same as in the base case.

The value of the revenue-based bond is the expected present value at time t_0:

$$B^{FRA}(R_{t_0}, t_0, t_1, t_N, s) = \mathbf{E}^Q\left[\sum_{i=1}^{N} e^{-\int_{t_0}^{t_i+s} r(s)ds} K_{t_i}\mathbf{1}_{\{\tau > t_i + s\}}\right],$$

and the explicit evaluation formula is:

$$\begin{aligned} B^{FRA}(R_{t_0}, t_0, t_1, t_N, s) = &\sum_{i=1}^{n^*}\Big[\big[H^{FRA}(t_0, t_i, t_i + s) \\ &- P(r, t_0, t_i + s)\mathcal{O}(V^*(t_i), R_{t_{i-1}}, t_0, t_i)\big]\mathbf{PF}(t_0, t_i)\mathbf{SP}(t_0, t_i + s)\Big] \\ &+ \Big[P(r, t_0, t_N + s)\mathbf{E}^Q[R_{t_{N-1}}] - \big[H^{FRA}(t_0, t_N, t_N + s) \\ &- P(r, t_0, t_N + s)\mathcal{O}(V^*(t_N), R_{t_{N-1}}, t_0, t_N)\big]\mathbf{PF}(t_0, t_i)\Big]\mathbf{SP}(t_0, t_N + s). \end{aligned} \tag{25}$$

3.3.2 Limited Recourse Clause

As before, only the case when invoices are assigned has to be considered.

Assignment of Invoices

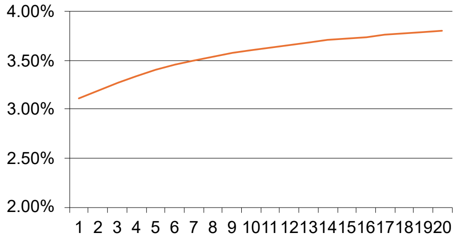
The stream of cash flows of a revenue-based bond is the sum of the fraction ω of future invoices issued in the reference periods and paid at the expiry dates, assumed to be a constant interval s after the ending dates $\{t_1, t_2, \dots, t_{n^*}, t_N\}$ of the reference periods. This time, we consider that the debtor company survives up to the ending dates of the reference periods $\{t_1, t_2, \dots, t_{n^*} + s, t_N\}$, and that the representative client company survives between each ending period and the payment date $[t_i, t_i + s]$, for $i \in \{1, \dots, n^*, N\}$.

Equations (17) and (22) are also used in this version of the RBF contract, whereas the value of the revenue-based bond is:

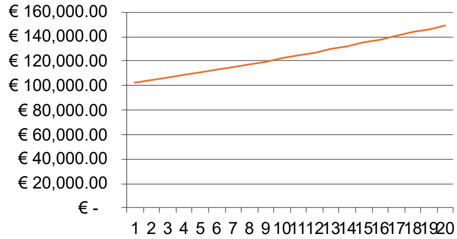
$$B^{LRA}(R_{t_0}, t_0, t_1, t_N, s) = \mathbf{E}^Q\left[\sum_{i=1}^{N} e^{-\int_{t_0}^{t_i+s} r(s)ds} K_{t_i}\mathbf{1}_{\{\tau > t_i\}}\mathbf{1}_{\{\tau^c > t_i + s | \tau^c > t_i\}}\right].$$

Interest Rate		Debtor's Default Intensity		Client's Default Intensity		Revenues	
r_0	3.00%	λ	3.00%	ξ	4.00%	$V(t_0)$	€ 100,000.00
κ_r	0.15	κ_λ	0.2	κ_ξ	0.2	μ	5.00%
θ_r	5%	θ_λ	3.50%	θ_ξ	5.00%	π_v	3.00%
σ_r	7.50%	σ_λ	6.00%	σ_ξ	6.00%	σ_v	3.0%

TABLE 3: Parameters of the risk factors' dynamics.



(a) Interest Rate Zero-Rate Curve.



(b) Evolution of expected revenues.

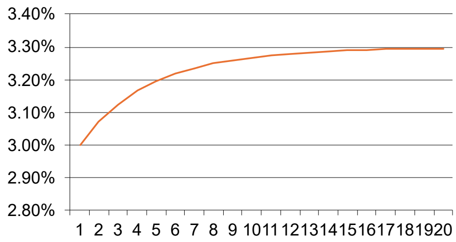
FIGURE 1: Interest rates and revenues.

The explicit value is then:

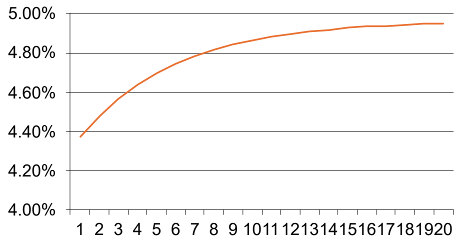
$$
\begin{aligned}
B^{LRA}(R_{t_0}, t_0, t_1, t_N, s) = &\sum_{i=1}^{n^*} \Big[[H^{LRA}(t_0, t_i, t_i + s) \\
&- P(r, t_0, t_i + s)\mathcal{O}(V^*(t_i), R_{t_{i-1}}, t_0, t_i)] \mathbf{PF}(t_0, t_i)\mathbf{SP}(t_0, t_i)\mathbf{SP}^c(t_i, t_i + s) \Big] \\
&+ P(r, t_0, t_N + s) \Big[\mathbf{E}^{\mathbb{Q}}[R_{t_{N-1}}] - [H^{LRA}(t_0, t_N, t_N + s) \\
&- \mathcal{O}(V^*(t_N), R_{t_{N-1}}, t_0, t_N)] \mathbf{PF}(t_0, t_i) \Big] \mathbf{SP}(t_0, t_N)\mathbf{SP}^c(t_N, t_N + s).
\end{aligned}
\tag{26}
$$

4. Application of the Framework

We apply the framework outlined above to a simulated environment. The parameters used for the dynamics of the risk factors introduced in Section 2 are shown in Table 3. Figure 1 shows the resulting risk-free zero-rate term structure and the evolution of the expected revenues over a 20-year horizon; Table 8 in Appendix A.2 shows also the values of the revenues generated by different values of the drift parameter μ. The term structures of the (conditional) 1-year default probability for the debtor company and the representative client company are shown in Figure 2. A final note on the market risk parameters: we set only the parameter for the revenues $\pi_V = 3.0\%$, whereas we assume that for all others risk factors they are equal to 0.
Given this set of parameters, we will now price the value of a revenue-based bond under different assumptions for the drift μ of the revenue dynamics in Equation (5). This represents the expected annual percentage variation of the revenues in the reference period. We start by analysing a revenue-based contract in the base case with full recourse: in Table 4, we show the main contract terms and the value of the revenue-based bond, the yield to maturity, the multiple of the initial lent amount (*i.e.*, the value of the bond) that must be repaid by the debtor, and the duration of the bond.



(a) One-year default rates' curve of the debtor company.



(b) One-year default rates' curve of the client company.

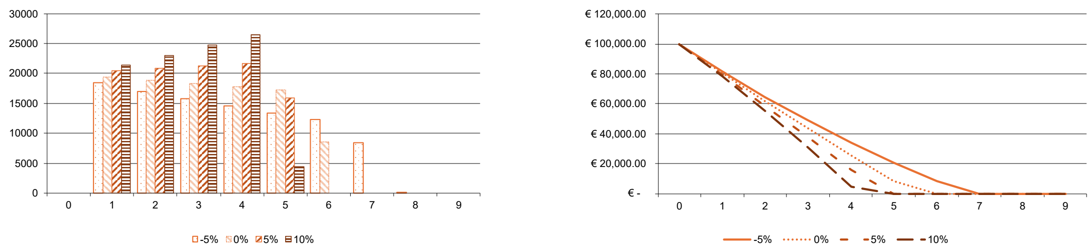
FIGURE 2: Default rates.

	μ			
	-5%	**0%**	**5%**	**10%**
V(t₀)	€ 100,000.00	€ 100,000.00	€ 100,000.00	€ 100,000.00
ω	20.00%	20.00%	20.00%	20.00%
Term Date	-	-	-	-
Floor	-	-	-	-
B	€ 79,853.80	€ 81,770.52	€ 83,237.13	€ 84,429.39
YTM	6.71%	6.67%	6.63%	6.60%
Multiple	1.25	1.22	1.20	1.18
Duration	3.34	3.03	2.79	2.60

TABLE 4: Contract terms and value of a revenue-based bond with risk and yield metrics in the base case contract with no term date and with no revenues' floor.

The terms of the contracts are the same under all possible values of the drift μ: the lender has the right to receive a fraction $\omega = 20\%$ of the revenues generated in each future reference period, assuming current revenues $V(t_0) = 100{,}000.00$. The contract provides for no term date and no floor on the revenues below which repayment does not occur.
The results show that the present value of the revenue-based bond (which is the fair amount that the lender should grant to the debtor company at the start of the revenue-based contract) increases with the value of the drift μ. On the other hand, the duration is inversely related to the level of μ, whereas the yield to maturity[2] and the multiple are quite stable. In Figure 3, the expected repayments K_{t_i} and the expected outstanding debt R_{t_i} are shown for the different levels of μ. In Table 9 in Appendix A.2, we show the values of the expected repayments and outstanding debt.
Next, we investigate the value of the revenue-based bond under different assumptions for the expected revenue growth rate μ, by adding a new term to the contract: a floor set at 80,000.00. If the revenues fall below this level, no repayment occurs. Table 5 and Figure 4 show the results as before.
It is interesting to analyse the case when the revenue growth rate is negative, -5.0%. As shown in Figure 4, and confirmed by Table 10 in Appendix A.2, the expected repayment stops after five years because expected revenues remain below the floor of 80,000.00.

[2] The yield to maturity is calculated in the usual way as the single rate that equates the (compound) discounted expected cash flows to the value of the bond.



(a) Expected repayments K_{t_i}. **(b)** Expected outstanding debt R_{t_i}.

FIGURE 3: Expected repayments and outstanding debt amounts for different levels of the drift parameter μ, in the base case contract with no term date and no revenues' floor.

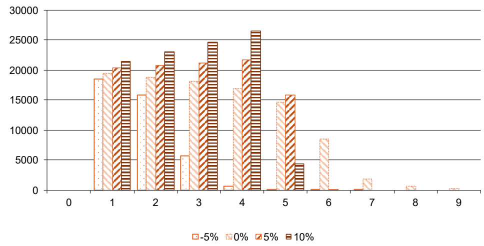
	μ			
	-5%	0%	5%	10%
$V(t_0)$	€ 100,000.00	€ 100,000.00	€ 100,000.00	€ 100,000.00
ω	20.00%	20.00%	20.00%	20.00%
Term Date	-	-	-	-
Floor	€ 80,000.00	€ 80,000.00	€ 80,000.00	€ 80,000.00
B	€ 36,638.30	€ 81,065.12	€ 83,237.13	€ 84,429.39
YTM	6.48%	6.68%	6.63%	6.60%
Multiple	2.73	1.23	1.20	1.18
Duration	1.69	3.08	2.79	2.60

TABLE 5: Contract terms and value of a revenue-based bond with risk and yield metrics in the base case contract with no term date and with a revenues' floor.

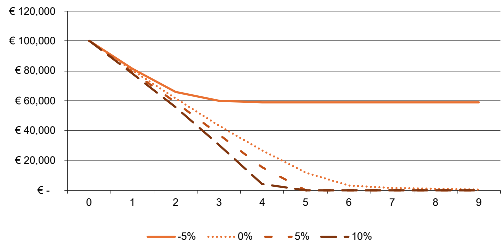
Consequently, the multiple of 2.73 on the value of the bond (36,638.30) is never fully repaid. Despite this partial recovery of the expected repaid amount (100,000.00), the lender does not suffer a negative return on the investment. In fact, it earns a total expected cash flow of 40,878.00, corresponding to an expected positive yield of 6.48%. The framework accounts for the decline in revenues and the floor, and sets the value of the revenue-based bond at 36,638.30, with a very high multiple. These evaluation outcomes make the investment profitable for the lender on an expectation basis.

Partial recovery of the multiple amount also occurs under the assumption of a revenue growth rate of 0.0%. In this case, the floor is never breached, so repayments never stop. Nonetheless, the percentage $\omega = 20\%$ of revenues, given their expected future evolution, is not sufficient to fully repay the multiple amount. Again, this does not imply that the lender's expected return on the investment is negative; on the contrary, the framework will provide results that keep practically constant the expected yield-to-maturity under all circumstances.

Finally, we examine the case where the contract provides for a term date of five years and a floor set at 80,000.00. Table 6 and Figure 5 show the results. The floor does not affect the full repayment in this case, since at the end of year 5 the entire outstanding debt must be reimbursed by the debtor company. The duration and the yield to maturity of the investment are not substantially different from the case where no term date and no floor applied. Thus, the introduction of a term date almost completely eliminates the effect



(a) Expected repayments K_{t_i}.



(b) Expected outstanding debt R_{t_i}.

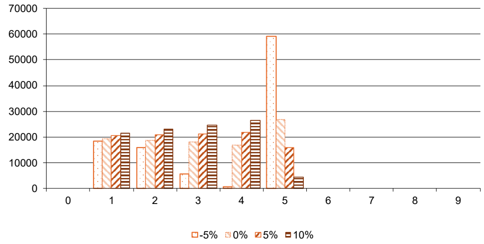
FIGURE 4: Expected repayments and outstanding debt amounts for different levels of the drift parameter μ, in the base case contract with no term date and with a revenues' floor.

	μ			
	-5%	**0%**	**5%**	**10%**
V(t₀)	€ 100,000.00	€ 100,000.00	€ 100,000.00	€ 100,000.00
ω	20.00%	20.00%	20.00%	20.00%
Term Date	5	5	5	5
Floor	€ 80,000.00	€ 80,000.00	€ 80,000.00	€ 80,000.00
B	€ 79,293.51	€ 82,132.20	€ 83,237.13	€ 84,429.39
YTM	6.71%	6.65%	6.63%	6.60%
Multiple	1.26	1.22	1.20	1.18
Duration	3.47	2.98	2.79	2.60

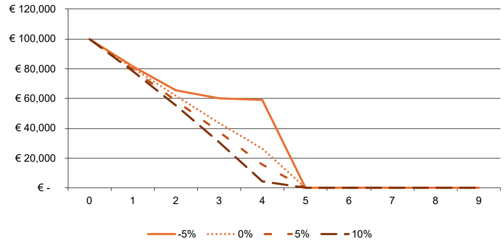
TABLE 6: Contract terms and value of a revenue-based bond with risk and yield metrics in the base case contract with a term date and with a revenues' floor.

of the floor on the risk and return metrics of the investment, although the floor may still be relevant to the debtor company as it can mitigate cash outflows in the event of steeply declining revenues. For the exact values of the expected repayments and outstanding amounts, see Table 11 in Appendix A.2.

The two remaining cases of contracts are left to analyse, *i.e.*, full recourse with assignment of invoices and limited recourse with assignment of invoices. In Table 7, we show the main results for the case where $\mu = 5.0\%$, and no term date or revenue floor applies. It is easy to verify that there is only a small difference in the value of the revenue-based bond, while the other metrics are almost identical across all contract types. We will not reproduce the detailed analysis for these two cases, as, all else being equal, the results are essentially the same as in the base case.



(a) Expected repayments K_{t_i}.



(b) Expected outstanding debt R_{t_i}.

FIGURE 5: Expected repayments and outstanding debt amounts for different levels of the drift parameter μ, in the base case contract with a term date and with a revenues' floor.

	Full Recourse	Full Rec. With Invoice Assig.	Limited Rec. with Invoice Assig.
V(t$_0$)	€ 100,000.00	€ 100,000.00	€ 100,000.00
ω	20.00%	20.00%	20.00%
Term Date	-	-	-
Floor	-	-	-
B	€ 83,237.13	€ 81,866.65	€ 81,585.42
YTM	6.60%	6.60%	6.60%
Multiple	1.20	1.22	1.23
Duration	2.79	2.79	2.79

TABLE 7: Contract terms and value of a revenue-based bond with risk and yield metrics for the three types of contracts.

5. Conclusions

We presented an evaluation framework for revenue-based contracts. We included several common contract terms observed in practice, and we also considered a less common case that we deem interesting: the assignment of invoices related to the revenues of the relevant reference period. The framework is sufficiently rich to capture the main risks in all possible variations of the contract terms, yet it remains tractable, with all formulae expressed in closed form.

The availability of a framework such as the one we presented allows for proper assessment of the financial and credit risks borne by the lender. This should also attract more investors to this alternative lending space, thereby enlarging funding opportunities for small and medium-sized enterprises.

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A. *Annex*

A.1 Proof of the Formulae for the Price of a Revenue-Based Zero-Coupon Bond

A.1.1 Full Recourse Clause

Base Case

The price at time t of a revenue-based zero-coupon bond $H(t,T)$ expiring in T, is the solution of the PDE (12) with terminal condition $H(V,r,\lambda,T,T) = \omega V(T)$. It is well known that the solution can be represented as an expectation[3] (under the risk-neutral measure Q):

$$H(V,r,\lambda,t,T) = \mathbf{E}^Q\left[e^{-\int_t^T r(s)ds}\omega V(T)\mathbf{1}_{\{\tau>T\}}\right].$$

For an explicit solution, let us try with a function of the type: $H(V,r,\lambda,t,T) = VP(r,t,T)N(\lambda,t,T)\omega$, with terminal condition $P(r,T,T) = 1$, $N(\lambda,T,T) = 1$. By replacing it in the PDE (12) and simplifying the notation, we get:

$$\begin{aligned}
&\frac{1}{2}\sigma_r^2 rP_{rr}VN\omega + \frac{1}{2}\sigma_\lambda^2\lambda N_{\lambda\lambda}VP\omega \\
&+(\mu-\pi_V)VNP\omega + [\kappa_r(\theta_r - r) - \pi_r rP_r VP\omega \\
&+[\kappa_\lambda(\theta_\lambda-\lambda) - \pi_\lambda\lambda]N_\lambda VP\omega + P_tVN\omega + N_tVP\omega \\
&-\lambda VPN\omega - rVPN\omega = 0.
\end{aligned} \tag{27}$$

We can split the PDE (34) in three PDE's, each one equation to 0. The first one collects all the terms where the risk factor λ is involved; after dividing by $VP\omega$ we get:

$$\frac{1}{2}\sigma_\lambda^2\lambda N_{\lambda\lambda} + [\kappa_\lambda(\theta_\lambda-\lambda) - \pi_\lambda\lambda]N_\lambda + N_t - \lambda N + (\mu-\pi_V)N = 0. \tag{28}$$

The solution is derived by imposing the terminal condition $N(\lambda,T,T) = 1$, so that as an expectation it is:

$$\begin{aligned}
N(\lambda,t,T) &= \mathbf{E}^Q\left[\mathbf{1}_{\tau>T}e^{-\int_t^T(\mu-\pi_V)ds}\right] \\
&= \mathbf{E}^Q\left[e^{-\int_t^T -\lambda(s)ds}\right]e^{(\mu-\pi_V)(T-t)}.
\end{aligned}$$

It is straightforward to note that the expectation is the price of a interest rate zero-coupon bond where the discounting is given by the process $\lambda(s)$. By exploiting the result in Cox, Ingersoll and Ross [3], and by considering that if λ follows the mean reverting square-root process (8), then the solution is:

$$N(\lambda,t,T) = C(t,T)e^{-\lambda(t)D(t,T)}e^{(\mu-\pi_V)(T-t)},$$

with $C(t,T)$ and $D(t,T)$ provided in the main text. We also stress that $N(t,T)$ takes into account the survival probability of the debtor company up to time T with the terms $\mathbf{SP}(t,T) = C(t,T)e^{-\lambda(t)D(t,T)}$; the second exponential $e^{(\mu-\pi_V)(T-t)}$ considers the remaining part of the drift of the revenues.

[3] See for example Friedman [5], Theorem 5.2.

The second PDE collects all the terms in the PDE (34) that involve the risk factor r; dividing by $VN\omega$ we get:

$$\frac{1}{2}\sigma_r^2 r P_{rr} + [\kappa_r(\theta_r - r) - \pi_r r]P_r + P_t - rP = 0, \tag{29}$$

with terminal condition $P(r,S,S) = 1$, whose solution as en expectation is:

$$P(r,t,T) = \mathbf{E}^Q\left[e^{-\int_t^T r(s)ds}\right].$$

The explicit solution is in the main text and it is the price of a zero-coupon bond provided in Cox, Ingersoll and Ross [3]. The explicit solution is in the main text.

Assignment of Invoices
The price at time t of a revenue-based zero-coupon bond $H(V,r,\lambda,t,T,S)$ expiring in S, when invoices issued in the reference period ending in T are paid, is the solution of the PDE (12) with terminal condition $H(V,r,\lambda,S,T,S) = \omega V(T)$. It is well known that the solution can be represented as an expectation[4])under the risk-neutral measure Q):

$$H(V,r,\lambda,t,T,S) = \mathbf{E}^Q\left[e^{-\int_t^S r(s)ds}\omega V(T)\mathbf{1}_{\{\tau>S\}}\right].$$

The explicit solution is very similar to the *Base case*, with the only difference given by the expiry of the discount factor and the survival of the debtor company up to the expiry S. The solution is

$$H(V,r,\lambda,t,T,S) = VP(r,t,S)\mathbf{SP}(t,S)e^{(\mu-\pi_V)(T-t)}\omega.$$

The proof follows the same steps as above, with the terminal conditions $P(r,S,S) = 1$, $N(\lambda,S,S) = 1$. In the main text the explicit formula is provided.

A.1.2 Limited Recourse Clause

Assignment of Invoices
The price at time t of a revenue-based zero-coupon bond $H(V,r,\lambda,T,T,S)$ expiring in S, when invoices issued in the reference period ending in T are paid, is the solution of the PDE (12) with terminal condition $H(V,r,\lambda,S,S,S) = \omega V(S)$. It is well known that the solution can be represented as an expectation[5])under the risk-neutral measure Q):

$$H(V,r,\lambda,t,T,S) = \mathbf{E}^Q\left[e^{-\int_t^S r(s)ds}\omega V(T)\mathbf{1}_{\{\tau^c>S|\tau^c>T\}}\right].$$

As a solution, let us try again with a function of the type: $H(V,r,\lambda,t,T,S) = VP(r,t,S)N(\lambda,t,T)$ $M(\xi,t,T,S)\omega$, with terminal conditions $P(r,S,S) = 1$, $N(\lambda,T,T) = 1$ and $M(\xi,t,S,S)$. By replacing it in the PDE (12) and simplifying the notation, we get:

$$\begin{aligned}
&\frac{1}{2}\sigma_r^2 r P_{rr} VNM\omega + \frac{1}{2}\sigma_\lambda^2 \lambda N_{\lambda\lambda} VPM\omega + \frac{1}{2}\sigma_\xi^2 \xi M_{\xi\xi} VPN \\
&+ (\mu - \pi_V)VNPM\omega + [\kappa_r(\theta_r - r) - \pi_r r P_r VPM\omega \\
&+ [\kappa_\lambda(\theta_\lambda - \lambda) - \pi_\lambda\lambda]N_\lambda VPM\omega + [\kappa_\xi(\theta_\xi - r) - \pi_\xi\xi]M_\xi VPN\omega \\
&+ P_t VNM\omega + N_t VPM\omega + M_t VPN\omega \\
&- \lambda VPNM\omega - \xi VPNM\omega - rVPNM\omega = 0.
\end{aligned} \tag{30}$$

[4]See for example Friedman [5], Theorem 5.2.
[5]See for example Friedman [5], Theorem 5.2.

We can split the PDE (34) in three PDE's, each one equation to 0. The first one collects all the terms where the risk factor λ is involved; after dividing by $VPM\omega$ we get:

$$\frac{1}{2}\sigma_\lambda^2\lambda N_{\lambda\lambda} + [\kappa_\lambda(\theta_\lambda - \lambda) - \pi_\lambda\lambda]N_\lambda + N_t - (1+\alpha)\lambda N + (\mu - \pi_V)N = 0. \tag{31}$$

The solution is derived by imposing the terminal condition $N(\lambda, T, T) = 1$, and it is the same as seen before in the *Base case*, with the explicit formula in the main text.
The second PDE collects all the terms in the PDE (34) that involve the risk factor r; dividing by $VMN\omega$ we get:

$$\frac{1}{2}\sigma_r^2 rP_{rr} + [\kappa_r(\theta_r - r) - \pi_r r]P_r + P_t - rP = 0, \tag{32}$$

with terminal condition $P(r, S, S) = 1$. Once again, the solution is tha same as above and the explicit formula provided in the main text.
The third PDE collects all the terms in the PDE (34) that involve the risk factor ξ, which after dividing by $VPN\omega$ we get:

$$\frac{1}{2}\sigma_\xi^2\xi M_{\xi\xi} + [\kappa_\xi(\theta_\xi - r) - \pi_\xi]M_\xi + M_t - \xi M = 0, \tag{33}$$

with terminal condition $M(\xi, S, S, S) = 1$. The solution expressed as an expectation is:

$$M(\xi, t, T, S) = \mathbf{E}^Q\left[\mathbf{1}_{\{\tau^c > S | \tau^c > T\}}\right] = \mathbf{E}^Q\left[e^{-\int_T^S \xi(s)ds}\right].$$

It should be noted that $M(\xi, t, T, S)$ is the survival probability of the generic client company that pays the invoice. As such, we do not refer to a specific company and we can be sure that there is always a surviving generic company whose invoices are transferred to the lender. That means that the default process is in practice memoryless and we do not need to condition on the survival of the company up to time T.
The solution is the price of a zero-coupon futures price, derived by Cox, Ingersoll and Ross [4] and explicitly provided in the main text.
In some types of contracts we are not considering a generic client but a specific company, so that the default process cannot memoryless: in this case need to calculate the survival probability up to time S conditioned to he survival up to time T, s that:

$$M(\xi, t, T, S) = \frac{\mathbf{SP}^c(t, S)}{\mathbf{SP}^c(t, T)}.$$

The solution for $\mathbf{SP}^c(t, S)$ is the same derived above for the survival probability of the debtor company $\mathbf{SP}(t, T)$, where parameters of the client's default intensity ξ replace those of the debtor's default intensity λ.
The price at time t of a revenue-based zero-coupon bond $H(V, r, \lambda, T, T, S)$ expiring in S, when invoices issued in the reference period ending in T are paid, is the solution of the PDE (12) with terminal condition $H(V, r, \lambda, S, S, S) = \omega V(S)$. It is well known that the solution can be represented as an expectation[6])under the risk-neutral measure Q):

$$H(V, r, \lambda, t, T, S) = \mathbf{E}^Q\left[e^{-\int_t^S r(s)ds}\omega V(T)\mathbf{1}_{\{\tau > S\}}\right].$$

The explicit solution is very similar to the *Base case*, with the only difference given by the expiry of the discount factor and the survival of the debtor company up to th

[6]See for example Friedman [5], Theorem 5.2.

expiry S. For an explicit solution, let us try with a function of the type: $H(t,T) = VP(r,t,T)N(\lambda,t,T)M(\xi,t,T,S)\omega$, with terminal condition $P(r,S,S) = 1$, $N(\lambda,T,T) = 1$ and $M(\xi,S,S,S) = 1$. By replacing it in the PDE (12) and simplifying the notation, we get:

$$\begin{aligned}
&\frac{1}{2}\sigma_r^2 rP_{rr}VNM\omega + \frac{1}{2}\sigma_\lambda^2\lambda N_{\lambda\lambda}VPM\omega + \frac{1}{2}\sigma_\xi^2\xi M_{\xi\xi}VPN\\
&+ (\mu-\pi_V)VNPM\omega + [\kappa_r(\theta_r - r) - \pi_r rP_rVPM\omega\\
&+ [\kappa_\lambda(\theta_\lambda-\lambda) - \pi_\lambda\lambda]N_\lambda VPM\omega + [\kappa_\xi(\theta_\xi - r) - \pi_\xi\xi]M_\xi VPN\omega\\
&+ P_tVNM\omega + N_tVPM\omega + M_tVPN\omega\\
&- \lambda VPNM\omega - \xi VPNM\omega - rVPNM\omega = 0.
\end{aligned} \tag{34}$$

We can split the PDE (34) in three PDE's, each one equation to 0. The first one collects all the terms where the risk factor λ is involved; after dividing by $VPM\omega$ we get:

$$\frac{1}{2}\sigma_\lambda^2\lambda N_{\lambda\lambda} + [\kappa_\lambda(\theta_\lambda-\lambda) - \pi_\lambda\lambda]N_\lambda + N_t - (1+\alpha)\lambda N + (\mu-\pi_V)N = 0. \tag{35}$$

The solution is derived by imposing the terminal condition $N(\lambda,T,T) = 1$, so that as an expectation it is:

$$\begin{aligned}
N(\lambda,t,T) &= \mathbf{E}^Q\left[\mathbf{1}_{\tau>T}e^{-\int_t^T(\mu-\pi_V)ds}\right]\\
&= \mathbf{E}^Q\left[e^{-\int_t^T-\lambda(s)ds}\right]e^{(\mu-\pi_V)(T-t)}.
\end{aligned}$$

It is straightforward to note that the expectation is the price of a interest rate zero-coupon bond where the discounting is given by the process $\lambda(s)$. By exploiting the result in Cox, Ingersoll and Ross [3], and by considering that if λ follows the mean reverting square-root process (8), then the solution is:

$$N(\lambda,t,T) = C(t,T)e^{-\lambda(t)D(t,T)}e^{(\mu-\pi_V)(T-t)},$$

with $C(t,T)$ and $D(t,T)$ provided in the main text. We also stress that $N(t,T)$ takes into account the survival probability of the debtor company up to time T with the terms $\mathbf{SP}(t,T) = C(t,T)e^{-\lambda(t)D(t,T)}$; the second exponential $e^{(\mu-\pi_V)(T-t)}$ considers the remaining part of the drift of the revenues.

The second PDE collects all the terms in the PDE (34) that involve the risk factor r; dividing by $VMN\omega$ we get:

$$\frac{1}{2}\sigma_r^2 rP_{rr} + [\kappa_r(\theta_r - r) - \pi_r r]P_r + P_t - rP = 0, \tag{36}$$

with terminal condition $P(r,S,S) = 1$, whose solution as en expectation is:

$$P(r,t,S) = \mathbf{E}^Q\left[e^{-\int_t^S r(s)ds}\right].$$

The explicit solution is in the main text and it is the price of a zero-coupon bond provided in Cox, Ingersoll and Ross [3].

The third PDE collects all the terms in the PDE (34) that involve the risk factor ξ, which after dividing by $VPN\omega$ we get:

$$\frac{1}{2}\sigma_\xi^2\xi M_{\xi\xi} + [\kappa_\xi(\theta_\xi - r) - \pi_\xi]M_\xi + M_t - \xi M = 0, \tag{37}$$

with terminal condition $M(\xi, S, S, S) = 1$. The solution expressed as an expectation is:

$$M(\xi, t, T, S) = \mathbf{E}^{Q}\left[\mathbf{1}_{\{\tau^c > S | \tau^c > T\}}\right] = \mathbf{E}^{Q}\left[e^{-\int_T^S \xi(s)ds}\right].$$

It should be noted that $M(\xi, t, T, S)$ is the survival probability of the generic client company that pays the invoice. As such, we do not refer to a specific company and we can be sure that there is always a surviving generic company whose invoices are transferred to the lender. That means that the default process is in practice memoryless and we do not need to condition on the survival of the company up to time T.

The solution is the price of a zero-coupon futures price, derived by Cox, Ingersoll and Ross [4] and explicitly provided in the main text.

In some types of contracts we are not considering a generic client but a specific company, so that the default process cannot memoryless: in this case need to calculate the survival probability up to time S conditioned to the survival up to time T, *i.e.*:

$$\mathbf{E}^{Q}\left[\mathbf{1}_{\{\tau^c > S | \tau^c > T\}}\right] = \mathbf{P}(\mathbf{1}_{\{\tau^c > S | \tau^c > T\}}) = \frac{\mathbf{P}(\mathbf{1}_{\{\tau^c > S\}})}{\mathbf{P}(\mathbf{1}_{\{\tau^c > T\}})} = \frac{\mathbf{E}^{Q}\left[\mathbf{1}_{\{\tau^c > S\}}\right]}{\mathbf{E}^{Q}\left[\mathbf{1}_{\{\tau^c > T\}}\right]},$$

so that:

$$M(\xi, t, T, S) = \frac{\mathbf{SP}^c(t, S)}{\mathbf{SP}^c(t, T)}.$$

The solution for $\mathbf{SP}^c(t, S)$ is the same derived above for the survival probability of the debtor company $\mathbf{SP}(t, T)$, where parameters of the client's default intensity ξ replace those of the debtor's default intensity λ.

A.2 Revenues, Expected Repayments and Oustanding Debts

	μ			
Time	**-5%**	**0%**	**5%**	**10%**
0	€100,000	€100,000	€100,000	€100,000
1	€92,312	€97,045	€102,020	€107,251
2	€85,214	€94,176	€104,081	€115,027
3	€78,663	€91,393	€106,184	€123,368
4	€72,615	€88,692	€108,329	€132,313
5	€67,032	€86,071	€110,517	€141,907
6	€61,878	€83,527	€112,750	€152,196
7	€57,121	€81,058	€115,027	€163,232
8	€52,729	€78,663	€117,351	€175,067
9	€48,675	€76,338	€119,722	€187,761
10	€44,933	€74,082	€122,140	€201,375
11	€41,478	€71,892	€124,608	€215,977
12	€38,289	€69,768	€127,125	€231,637
13	€35,345	€67,706	€129,693	€248,432
14	€32,628	€65,705	€132,313	€266,446
15	€30,119	€63,763	€134,986	€285,765
16	€27,804	€61,878	€137,713	€306,485
17	€25,666	€60,050	€140,495	€328,708
18	€23,693	€58,275	€143,333	€352,542
19	€21,871	€56,553	€146,228	€378,104
20	€20,190	€54,881	€149,182	€405,520

TABLE 8: Projection of expected revenues.

	μ							
	-5%		0%		5%		10%	
Time	K	R	K	R	K	R	K	R
0		€100,000		€100,000		€100,000		€100,000
1	€18,462	€81,538	€19,409	€80,591	€20,404	€79,596	€21,450	€78,550
2	€17,043	€64,495	€18,835	€61,756	€20,816	€58,780	€23,005	€55,544
3	€15,733	€48,762	€18,279	€43,477	€21,237	€37,543	€24,674	€30,871
4	€14,523	€34,239	€17,738	€25,739	€21,666	€15,877	€26,460	€4,411
5	€13,406	€20,833	€17,214	€8,525	€15,877	€0	€4,411	€-
6	€12,376	€8,457	€8,525	€0	€-	€0	€-	€-
7	€8,457	€0	€-	€0	€-	€0	€-	€-
8	€0	€0	€-	€0	€-	€0	€-	€-
9	€-	€0	€-	€0	€-	€0	€-	€-

TABLE 9: *Expected repayments and outstanding debt amounts for different levels of the drift parameter μ, in the base case contract with no term date and no revenues' floor.*

μ								
	-5%		0%		5%		10%	
Time	**K**	**R**	**K**	**R**	**K**	**R**	**K**	**R**
0		€100,000		€100,000		€100,000		€100,000
1	€18,462	€81,538	€19,409	€80,591	€20,404	€79,596	€21,450	€78,550
2	€15,830	€65,708	€18,834	€61,757	€20,816	€58,780	€23,005	€55,544
3	€5,711	€59,997	€18,176	€43,581	€21,237	€37,543	€24,674	€30,871
4	€727	€59,270	€16,929	€26,651	€21,666	€15,877	€26,460	€4,411
5	€51	€59,219	€14,713	€11,938	€15,877	€0	€4,411	€-
6	€3	€59,216	€8,464	€3,474	€0	€0	€-	€-
7	€0	€59,216	€1,911	€1,563	€-	€0	€-	€-
8	€0	€59,216	€633	€930	€-	€0	€-	€-
9	€0	€59,216	€266	€665	€-	€0	€-	€-
10	€0	€59,216	€130	€535	€-	€0	€-	€-
11	€0	€59,216	€70	€465	€-	€0	€-	€-
12	€0	€59,216	€40	€425	€-	€0	€-	€-
13	€0	€59,216	€23	€402	€-	€0	€-	€-
14	€0	€59,216	€14	€387	€-	€0	€-	€-
15	€-	€59,216	€9	€379	€-	€0	€-	€-
16	€-	€59,216	€5	€374	€-	€0	€-	€-
17	€-	€59,216	€3	€370	€-	€0	€-	€-
18	€-	€59,216	€2	€368	€-	€0	€-	€-
19	€-	€59,216	€1	€367	€-	€0	€-	€-
20	€-	€59,216	€1	€366	€-	€0	€-	€-

TABLE 10: Expected repayments and outstanding debt amounts for different levels of the drift parameter μ, in the base case contract with no term date and with a revenues' floor.

μ								
	-5%		0%		5%		10%	
Time	K	R	K	R	K	R	K	R
0		€100,000		€100,000		€100,000		€100,000
1	€18,462	€81,538	€19,409	€80,591	€20,404	€79,596	€21,450	€78,550
2	€15,830	€65,708	€18,834	€61,757	€20,816	€58,780	€23,005	€55,544
3	€5,711	€59,997	€18,176	€43,581	€21,237	€37,543	€24,674	€30,871
4	€727	€59,270	€16,929	€26,651	€21,666	€15,877	€26,460	€4,411
5	€59,270	€-	€26,651	€-	€15,877	€-	€4,411	€-
6	€-	€-	€-	€-	€-	€-	€-	€-
7	€-	€-	€-	€-	€-	€-	€-	€-
8	€-	€-	€-	€-	€-	€-	€-	€-
9	€-	€-	€-	€-	€-	€-	€-	€-
10	€-	€-	€-	€-	€-	€-	€-	€-

TABLE 11: Expected repayments and outstanding debt amounts for different levels of the drift parameter μ, in the base case contract with a term date and with a revenues' floor.

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