Research Paper Series

A Common Collateral Pooling Arrangement for Corporate Lending

Antonio Castagna

September 2025

iason
ESSENTIAL SERVICES FOR
FINANCIAL INSTITUTIONS

iason
Research Paper Series
Year 2025 - Issue Number 79

Last published issues are available online:
http://www.iasonltd.com/research

Front Cover: **Atanasio Soldati**, *Untitled*, 1936.

Executive Summary

We present a framework for establishing a mutual collateral scheme that a group of corporate debtors can use to enhance the overall credit quality of the loan portfolio extended to them by an investor. We analyze the extent of credit protection provided by the pooled collateral and how the debtors can be compensated for it, based on actuarial and financial fairness principles.

About the Author

Antonio Castagna:
Founder and Managing Partner
Antonio Castagna is currently managing partner and founder of the consulting company iason. He previously was in Banca IMI, Milan, from the 1999 to 2006: there, he first worked as a market maker of cap/floor's and swaptions; then he set up the FX options desk and ran the book of plain vanilla and exotic options on the major currencies, being also responsible for the entire FX volatility trading. He started his carrier in the investment banking in the 1997 in IMI Bank, in Luxemborug, as a financial analyst in the Risk Control Department. He graduated in Finance at LUISS University in Rome in 1995, with a thesis on American options and the numerical procedures for their valuation. He wrote papers on different topics, including credit risk, derivative pricing, collateral management, managing of exotic options risks and volatility smiles. He is also author of the books "FX options and smile risk" and "Measuring and Managing Liquidity Risk", both published by Wiley.

Table of Content

A Common Collateral Pooling Arrangement for Corporate Lending

Antonio Castagna

We analysed in a previous paper (see Castagna [3]) a risk-sharing mechanism for a group of companies selling their accounts receivable to an investor. In this work, we extend the idea by analysing a collateral pooling agreement that allows part of the investorâs credit risk to be covered. The agreement should facilitate loan approval decisions and potentially lead to an enhancement of the interest rate paid by the borrowers.
The concept hinges on the old idea of mutual aid, *i.e.*: an arrangement where resources are voluntarily shared for the mutual benefit of a group of peers. Contemporary forms of mutual aid can be more easily implemented through modern technologies that enable effective organisation and operation of such schemes, often referred to as P2P risk sharing.
Current research on P2P risk sharing includes Abdikerimova and Feng [1], who analyse the concept of P2P risk transfer and allocation networks; Denuit and Dhaene [4], who introduce a conditional mean risk-sharing mechanism to achieve Pareto-optimality by leveraging the risk-reducing properties of conditional expectations with respect to convex ordering; building on this approach, Denuit and Robert [5] formalise the three business models dominating peer-to-peer (P2P) property and casualty insurance: the self-governing model, the broker model, and the carrier model; Feng et al. [9] propose a framework for P2P risk-sharing pools based on Pareto optimality and actuarial fairness principles, providing an exact solution for allocation ratios in the unconstrained optimal P2P risk-sharing setting, with an application to catastrophe risk pooling and several P2P alternatives for flood risk management.
The work is organised as follows: we present the general setup of a portfolio of loans and the contract terms they share; then, we analyse how collateral can be posted by the borrowers and its dynamics, also considering the credit losses suffered by the lender. We then study how the premium for the protection offered by the pooled collateral is determined, and how this premium should be allocated to individual borrowers based on actuarial and financial fairness conditions. Finally, we investigate how to calculate the premium through a numerical procedure and present a practical implementation of the scheme.

1. Credit Risk and Collateral Agreement

Let us assume that at time $t = 0$, a group of companies seeks financing from one or more lending entities. Each company i ($i \in \{1, \ldots, I\}$) requires funding for a notional amount $K_i(0)$, with maturity at $t = T$, and agrees to pay an annual interest rate r_i. We assume that the maturity T is common to all companies; a more general case will be addressed later in this work.
The interval $[0, T]$ is divided into discrete time points at which interest and principal payments occur, *i.e.*, $t \in \{0, 1, \ldots, T\}$.
Each loan is repaid according to an amortisation schedule $A_i(a)$, with $a \in \{1, \ldots, T\}$, such that:

$$\sum_{a=1}^{T} A_i(a) = K_i(0).$$

The outstanding amount at any time t is then defined as:

$$K_i(t) = K_i(0) - \sum_{a \le t} A_i(a).$$

The debtor companies are subject to possible default: let τ_i be the time of default of debtor i. If $t \le \tau_i \le T$, the lender suffers a loss $L_i(\tau_i)$, which is a percentage $\mathbf{Lgd}_i$ (assumed fixed) of the exposure at default $\mathbf{EAD}_i(\tau_i)$, *i.e.*, the outstanding amount of the loan at time τ_i. Thus:[1]

$$L_i(\tau_i) = \mathbf{EAD}_i(\tau_i) \times \mathbf{Lgd}_i = \left[K_i(0) - \sum_{a=1}^{\tau_i - 1} A_i(a) \right] \times \mathbf{Lgd}_i. \tag{1}$$

Let $D_i(t,s) \in \{0,1\}$ be the indicator function of default ($D_i(t,s) = 1$) for debtor i between times t and s ($t < s$). The probability that debtor i defaults between t and s is denoted by $\mathbf{PD}_i(t,s)$, which may be abbreviated as $\mathbf{PD}_i$ when the context clearly refers to the interval $[0,T]$. The corresponding survival probability is $\mathbf{SP}_i(t,s) = 1 - \mathbf{PD}_i(t,s)$. Although default can occur at any instant τ_i, it is only observed at discrete times within the interval $[0,T]$. We assume that defaults among debtors are independently distributed events.
The credit risk borne by the lender can complicate the borrowing process, both in terms of the amount actually lent - which may be less than needed - and in terms of the interest rate required to compensate for the risk. To mitigate this issue, the debtor companies agree to post collateral to reduce the lender's credit risk. To minimise the total collateral posted and maximise its effectiveness in reducing credit risk, the companies agree to pool the collateral contributed by each of them and use it to cover any credit losses suffered by the lender, regardless of which companies actually default.

1.1 Collateral Dynamics and Net Credit Loss

Depending on the arrangement between the debtor companies and the lender(s), we can determine the dynamics of the collateral pool, assuming we are at time $t = 0$. In the most general agreement, each company i agrees to deposit as collateral an amount of cash C_i equal to a percentage x_i of the initial borrowed amount $K_i(0)$. The deposit is made in $M \le T$ instalments $C_i^q(t)$ at one or more times t, so that the total collateral contributed by company i at any time $t \ge 0$ is:

$$C_i(t) = \sum_{m=0}^{t \vee M} C_i^q(m) \cdot (1 - D_i(0,m)). \tag{2}$$

The collateral contributed by company i depends on the accumulation plan and the survival of the company up to each instalment time m. If company i survives up to the last instalment time M, then for any time $t > M$ we have:

$$C_i(t) = \sum_{m=0}^{M} C_i^q(m) = x_i K_i(0).$$

In a typical arrangement, the accumulation schedule is the same for all debtors, and the total collateral contributed by each of them is a common percentage of the borrowed amount.
The expected collateral contributed up to time t, calculated at time 0, is:

$$EC_i(t) = \mathbb{E}[C_i(t)] = \sum_{m=0}^{M} C_i^q(m) \cdot \mathbf{SP}_i(0,m). \tag{3}$$

The pooled collateral $C_P(t)$ contributed by all debtor companies is kept in an escrow account until maturity T. At any time t, it is simply the sum of the collateral posted by each company:

$$C_P(t) = \sum_{i=1}^{I} C_i(t). \tag{4}$$

[1] We do not include in the credit loss the possible missed interest payments due from the debtor. These can be easily incorporated into the analysis without substantially altering the structure of the arrangement, aside from the definition of credit loss.

Similarly, the expected pooled collateral at time t, calculated at time 0, is:

$$EC_P(t) = \mathbb{E}\left[\sum_{i=1}^{I} C_i(t)\right].$$

Over the interval $[0, T]$, the cumulative total credit loss suffered by the lender is:

$$L_P(T) = \sum_{i=1}^{I}\sum_{t=1}^{T} L_i(t) \cdot D_i(t-1,t). \tag{5}$$

The expected credit loss $EL_P(T)$ is then:

$$EL_P(T) = \mathbb{E}[L_P(T)] = \sum_{i=1}^{I}\sum_{t=0}^{T} L_i(t) \cdot \mathbf{PD}_i(t-1,t). \tag{6}$$

This loss can be mitigated by the available collateral.
It is also useful to compute the present value of $EL_P(T)$. In general terms, let $\mathcal{D}(t,T)$ be the discount factor used to compute the present value at time t of a cash flow occurring at time T. Denoting by r_t the instantaneous risk-free interest rate at time t, we have:

$$\mathcal{D}(t,T) = \exp\left(-\int_t^T r_s\, ds\right).$$

In a stochastic interest rate framework, the discount factor is defined as:

$$P(t,T) = \mathbb{E}^Q\left[\mathcal{D}(t,T)\right] = \mathbb{E}^Q\left[\exp\left(-\int_t^T r_s\, ds\right)\right],$$

where the expectation is taken under the risk-neutral measure Q. We also assume that r_s is independent of any other stochastic variable in the model we are presenting.
The present value of the expected total credit loss is:

$$EL_P^D(T) = \mathbb{E}\left[\sum_{i=1}^{I}\sum_{t=0}^{T} \mathcal{D}(0,t)L_i(t)D_i(t-1,t)\right] = \sum_{i=1}^{I}\sum_{t=0}^{T} P(0,t)L_i(t)\mathbf{PD}_i(t-1,t). \tag{7}$$

Whenever a debtor defaults, the collateral is used to compensate the lender for the outstanding amount of the loan, *i.e.*, $\mathbf{EAD}_i(t) = K_i(0) - \sum_{a<t} A_i(a)$. The credit loss suffered by the lender upon the default of debtor i is zero only if a sufficient residual amount of pooled collateral is available, considering previous defaults. Since the collateral is accumulated according to a predefined schedule, it can be used to cover both future and past credit losses.
At time T, the present value of the cumulative total credit loss suffered by the lender, net of the protection offered by the pooled collateral $C_P(T)$, denoted by $NL_P^D(T)$, is:

$$NL_P^D(T) = \left[\sum_{i=1}^{I}\sum_{t=1}^{T} \mathcal{D}(0,t)L_i(t)D_i(t-1,t) - \sum_{t=1}^{T}\mathcal{D}(0,t)\Delta R(t)\right], \tag{8}$$

where $\Delta R(t) = \min[L_P(t), C_P(t)] - \min[L_P(t-1), C_P(t-1)]$ represents the compensation for credit losses at each time t, including possible uncovered past losses. The protection from credit losses is capped at the available pooled collateral, which may be fully exhausted by time T, or partially used, leaving residual collateral to be redistributed to the debtor companies according to a predefined rule.
The collateral can be interpreted as a form of insurance against credit losses, with a cap at the total pooled amount funded according to the instalment plan set at time 0. The expected value of the present value of the net loss is:

$$ENL_P(T) = \mathbb{E}[NL_P^D(T)] = EL_P^D(T) - \mathbb{E}\left[\sum_{t=1}^{T}\mathcal{D}(0,t)\Delta R(t)\right]. \tag{9}$$

The first term on the right-hand side of Equation (9) is the expected loss of the loan portfolio; the second term is the expected coverage provided by the available collateral, capped at its total pooled value.
Under fair actuarial principles, the premium Π that the lender would pay to purchase the protection offered by the pooled collateral arrangement equals the expected present value of the credit loss coverage:

$$\Pi = \sum_{t=1}^{T} P(0,t) \cdot \mathbb{E}\left[\min[L_P(t), C_P(t)] - \min[L_P(t-1), C_P(t-1)]\right]. \tag{10}$$

This premium should be paid to the individual debtors and depends, in addition to the planned total amount to be posted, also on the probability that the pooled collateral is not fully contributed by each company, in cases where the collateral is posted according to a schedule rather than upfront. There is a risk that a debtor may default before contributing the full amount. Possible approaches to the valuation of Π are presented in the following sections.

2. Fair Allocation of the Premium and of the Remaining Collateral

Since the pooled collateral can be interpreted as the full funding of a capped insurance against credit losses suffered by the lender, debtor companies should be fairly remunerated for their expected contribution to covering actual losses (within the limit of the posted collateral). In this section, we outline the rules for making the entire pooled collateral agreement fair for all debtors.
We consider a setup in which every debtor company participates in the reimbursement of the remaining collateral pool, regardless of whether it defaulted before time T. This setup allows for a tractable - though not closed-form - solution to the problem.
Let us consider company i: it contributes an expected amount $\mathbb{E}[C_i(T)]$ to the collateral pool according to the contribution schedule. For this contribution, it should receive a fraction $0 \leq \alpha_i \leq 1$ (with $\sum_i \alpha_i = 1$) of the total insurance premium Π paid by the lender for the credit protection provided by the collateral pool. Additionally, if some collateral remains after covering the credit losses at time T, company i receives a fraction $0 \leq \beta_i \leq 1$ of this residual amount.
At maturity T, the total outcome for company i resulting from the collateral agreement is:

$$R_i^A(T) = \alpha_i \Pi + \beta_i \max\left[\sum_{i=1}^{I}\sum_{t=1}^{T} C_i(t) - \sum_{i=1}^{I}\sum_{t=1}^{T} L_i(t) D_i(t-1,t), 0\right] - C_i(T). \tag{11}$$

The quantities α_i and β_i represent what company i receives, respectively, as a fraction of the premium and of the residual collateral after the credit losses are deducted (this quantity is clearly floored at zero), while $C_i(T)$ is its actual contribution to the collateral pool. It should be noted that Equation (11) does not include any discounting of cash flows and should be interpreted purely from an actuarial fairness perspective.
We can rewrite the argument of the max operator in Equation (11) as:

$$\max\left[\sum_{i=1}^{I}\sum_{t=1}^{T} C_i(t) - \sum_{i=1}^{I}\sum_{t=1}^{T} L_i(t) D_i(t-1,t), 0\right] = \left[\sum_{i=1}^{I}\sum_{t=1}^{T} C_i(t) - \min\left[\sum_{i=1}^{I}\sum_{t=1}^{T} L_i(t) D_i(t-1,t), \sum_{i=1}^{I}\sum_{t=1}^{T} C_i(t)\right]\right].$$

Taking the expected value of $R_i^A(T)$, and using Equation (10), we obtain:

$$\mathbb{E}[R_i^A(T)] = \alpha_i \Pi + \beta_i \left[\mathbb{E}[C_P(T)] - \Pi\right] - \mathbb{E}[C_i(T)]. \tag{12}$$

Equation (12) explicitly shows the expected net collateral return (the term in square brackets) as the difference between the expected collateral pool at time T and the expected credit losses, which equal the fair premium Π. Actuarial fairness requires that $\mathbb{E}[R_i^A(T)] = 0$, and thus we must define rules for setting α_i and β_i to satisfy this condition.

Let us start with the share of residual pooled collateral β_i. The rule we aim to define must satisfy the following constraints:

$$0 \leq \beta_i \leq 1,$$
$$\sum_i \beta_i = 1.$$

so that the residual collateral is fully returned to the debtor companies, and none of them is required to make any additional payment. A simple and sensible rule is to return to each company a share proportional to its expected contribution to the collateral pool up to time T, *i.e.*,

$$\beta_i = \beta_i^* = \frac{EC_i(T)}{EC_P(T)}. \tag{13}$$

It is easy to verify that this rule satisfies both constraints.
Plugging Equation (13) into Equation (12), and computing the expected value, we obtain:

$$\mathbb{E}[R_i^A(T)] = \alpha_i \Pi - \frac{EC_i(T)}{EC_P(T)} \Pi. \tag{14}$$

From Equation (14), the fair value of α_i that ensures $\mathbb{E}[R_i^A(T)] = 0$ is:

$$\alpha_i = \alpha_i^* = \frac{EC_i(T)}{EC_P(T)}. \tag{15}$$

This choice also satisfies the constraints on α_i.
While these conditions ensure actuarial fairness, they do not necessarily guarantee financial fairness. To address this, we must consider the timing of the various cash flows and apply appropriate discounting.
Discounting the cash flows in Equation (11) can be done under different assumptions regarding when the premium Π is paid to each borrower. Without loss of generality, we assume the premium is paid in instalments over the dates $t \in \{0, \ldots, T\}$, with each instalment equal to a fraction ξ of Π. The discounted return from the collateral agreement is then:

$$R_i^F(T) = \alpha_i^* \xi \Pi \sum_{t=0}^{T} \mathcal{D}(0,t) + \beta_i^* \mathcal{D}(0,T) \left[C_P(T) - \Pi\right] - \sum_{m=0}^{M} \mathcal{D}(0,m) C_i^q(m)(1 - D_i(0,m)). \tag{16}$$

Taking expectations, we obtain:

$$\mathbb{E}[R_i^F(T)] = \alpha_i^* \xi \Pi \sum_{t=0}^{T} P(0,t) + \beta_i^* P(0,T) \left[EC_P(T) - \Pi\right] - \sum_{m=0}^{M} P(0,m) C_i^q(m) \cdot SP_i(0,m), \tag{17}$$

where ξ is such that $\alpha_i^* \xi \Pi \sum_{t=0}^{T} P(0,t) = \alpha_i^* \Pi$. The introduction of discounting generally implies $\mathbb{E}[R_i^F(T)] \neq 0$, so we introduce an additional payment Φ_i from the lender to each borrower, paid with the same schedule and fraction ξ as the premium Π, to restore financial fairness. That is:

$$\mathbb{E}[R_i^F(T)] = \alpha_i^* \xi (\Pi + \Phi_i) \sum_{t=0}^{T} P(0,t) + \beta_i^* P(0,T) \left[EC_P(T) - \Pi\right] - \sum_{m=0}^{M} P(0,m) C_i^q(m) \cdot SP_i(0,m) = 0. \tag{18}$$

Let $EC_i^D = \sum_{m=0}^{M} P(0,m) C_i^q(m) \cdot SP_i(0,m)$. Then the additional payment is making nil the financial return $R_i^F(T)$ is:

$$\Phi_i = \Phi_i^* = \frac{EC_i^D - \beta_i^* P(0,T) \left[EC_P(T) - \Pi\right] - \alpha_i^* \xi \Pi \sum_{t=0}^{T} P(0,t)}{\alpha_i^* \xi \sum_{t=0}^{T} P(0,t)}. \tag{19}$$

This value will typically be negative. These payments by the lender may be netted, on each date, with the payments the debtor must make for interest and amortisation of the loan.

3. Application of the Framework to a Homogeneous Portfolio

We implement the model under the assumption of a homogeneous portfolio of debtor companies and loans. More realistic assumptions will be addressed later by approximating the formulae derived for the homogeneous case.

This portfolio consists of I debtor companies, each with an identical probability of default, *i.e.*, $\mathbf{PD}_i(t,s) = \mathbf{PD}(t,s)$ for $s \in [0,T]$, $t \leq s$. It is important to stress that all defaults are independent events.

Additionally, each company borrows the same initial amount $K_i(0) = K(0)$, follows the same amortisation schedule $\sum_{a=0}^{T} A_i(a) = \sum_{a=0}^{T} A(a)$, and has the same loss given default, so that $L_i(\tau_i) = L(\tau_i) = \mathbf{EAD}(\tau_i) \times \mathbf{Lgd}$. All companies also post the same amount of collateral according to the same payment schedule, *i.e.*, $C_i(m) = x_i K_i(0) = C(m) = xK(0)$, with $\sum_{m=0}^{M} C_i^q(m) = \sum_{m=0}^{M} C^q(m) = xK(0)$.

Under these assumptions, the number of defaults between times t and s follows a binomial distribution $\mathcal{B}_{t,s}(I, \mathbf{PD}(t,s))$, with I trials. The cumulative total credit loss distribution is directly linked to the default distribution. At any time t, the default of a company in the interval $[0,t]$ produces a loss equal to the fraction of the outstanding portfolio $L(\tau_i)$, due to the homogeneity of the loans and amortisation schedules.

The total credit loss density function between $t-1$ and t is:

$$L_P(t-1,t) = L(t) \times \mathcal{B}'(i, I, \mathbf{PD}(t-1,t)), \tag{20}$$

where $\mathcal{B}'(i, I, \mathbf{PD})$ is the binomial density function evaluated at i, *i.e.*, the probability that exactly i defaults occur.

The expected credit loss is:

$$EL_P(t-1,t) = \mathbb{E}[L_P(t-1,t)] = \sum_{i=1}^{I} i \times L(t) \times \mathcal{B}'(i, I, \mathbf{PD}(t-1,t)) = I \times L(t) \times \mathbf{PD}(t-1,t). \tag{21}$$

The standard deviation (volatility) of the total credit loss is:

$$\mathbf{Vol}(L_P(t-1,t)) = I \times L^2(t) \times \mathbf{PD}(t-1,t)(1 - \mathbf{PD}(t-1,t)). \tag{22}$$

Unfortunately, even under the homogeneous portfolio assumption, the premium Π (*i.e.*, the expected protection offered by the collateral pool) cannot be computed in closed form. We propose a numerical scheme inspired by methods from Duffie and Garleanu [7], Duffie and Singleton [8], Duffie [6], and Brigo, Pallavicini, and Torresetti [2]. The procedure exploits the homogeneous portfolio assumption:

Procedure 3.1 (Premium Valuation under Homogeneous Portfolio)**.**

1. *Let S be the total number of simulations and $sd(t)$ the number of surviving debtors at time t.*
2. *For each of the S simulations, set $sd(0) = I$, $C_P(0) = sd(0) \times C^q(0)$, $L_P(0) = 0$, $\Pi_s(t) = 0$, and $\Pi_p = 0$.*
3. *For each time $t \in \{1, \ldots, T\}$, draw the number of defaults $nd(t)$ from a binomial distribution $\mathcal{B}(sd(t-1), \mathbf{PD}(t-1,t))$.*
4. *Update the total credit loss: $L_P(t) = L_P(t-1) + nd(t) \times \mathbf{Lgd} \times \mathbf{EAD}(t-1)$.*
5. *Update the number of surviving debtors: $sd(t) = sd(t-1) - nd(t)$.*
6. *Update the posted collateral: $C_P(t) = C_P(t) + sd(t) \times C^q(t)$.*
7. *Update the premium:*

$$\Pi_s(t) = \Pi_s(t-1) + P(0,t)\left[\min(L_P(t), C_P(t)) - \min(L_P(t-1), C_P(t-1))\right].$$

8. *Store the premium: $\Pi_p = \Pi_p + \Pi_s(T)$.*

9. *After completing all S simulations, compute the premium:* $\Pi = \Pi_p / S$.

Although the premium is computed numerically, the homogeneous portfolio assumption makes Procedure 3.1 efficient. If the portfolio is not homogeneous, this procedure cannot be applied directly, and a more general numerical scheme must be designed (see Duffie and Singleton [8], Duffie [6]). In the next section, we explore an approximation method to adapt a non-homogeneous portfolio to a homogeneous one, allowing continued use of Procedure 3.1.

4. Approximation to a Homogeneous Portfolio

We resort to an approximation that makes the actual portfolio as similar as possible to an equivalent homogeneous portfolio.[2] This approach allows us to derive, for each time $t \in \{1, \ldots, T\}$, the equivalent default probability $\mathbf{PD}^*(t-1,t)$, the outstanding loan amount $\mathbf{EAD}^*(t)$, and the equivalent number of debtors $I^*(t)$ of a homogeneous portfolio that approximates the actual one, assuming that the loss given default is the same for all debtors, *i.e.*, $\mathbf{Lgd}_i = \mathbf{Lgd}$.
We apply a Moment Matching Technique (MMT), which equates the first and second moments of the credit loss, *i.e.*: the expected value and variance of the actual portfolio over the period $[t-1,t]$, to those of the equivalent homogeneous portfolio. This is done under the constraint that the total value of the outstanding loans in both portfolios is the same. Formally:

$$\begin{cases} \mathbb{E}[L(t-1,t)] = \sum_{i=1}^{I} L_i(t) \cdot \mathbf{PD}_i(t-1,t) = I^*(t) \cdot L \cdot \mathbf{PD}^*(t-1,t) \\ \mathrm{Var}[L(t-1,t)] = \sum_{i=1}^{I} (L_i^*(t))^2 \cdot \mathbf{PD}_i(t-1,t) \cdot (1-\mathbf{PD}_i(t-1,t)) \\ \qquad = I^*(t) \cdot (L_i^*(t))^2 \cdot \mathbf{PD}^*(t-1,t) \cdot (1-\mathbf{PD}^*(t-1,t)) \\ I^*(t) \cdot \mathbf{EAD}^*(t) = K \end{cases}, \tag{23}$$

where $K = \sum_i \mathbf{EAD}_i(t)$ is the total value of the outstanding loans at time t.
The solution to the system (23) is:

$$\begin{cases} \mathbf{PD}^*(t-1,t) = \dfrac{\sum_{i=1}^{I} \mathbf{EAD}_i(t) \cdot \mathbf{PD}_i(t-1,t)}{K} \\ \mathbf{EAD}^*(t) = \dfrac{\sum_{i=1}^{I} \mathbf{EAD}_i^2(t) \cdot \mathbf{PD}_i(t-1,t) \cdot (1-\mathbf{PD}_i(t-1,t))}{K \cdot \mathbf{PD}^*(t-1,t) \cdot (1-\mathbf{PD}^*(t-1,t))} \\ I^*(t) = \dfrac{K}{\mathbf{EAD}^*(t)} \end{cases}. \tag{24}$$

It is important to note that this approximation is used only to compute the premium in Equation (10), while all other quantities must be computed using the original input data.
Procedure 3.1 must be adapted to carefully handle the number of debtors and the posting of collateral. We propose the following procedure, which extends the original one designed for the homogeneous portfolio:

Procedure 4.1 (Premium Valuation under Approximated Homogeneous Portfolio)**.**

1. *For each time* $t \in \{1, \ldots, T\}$, *compute* $\mathbf{PD}^*(t-1,t)$, $\mathbf{EAD}^*(t)$, *and* $I^*(t)$ *from the actual data.*

2. *From the actual collateral instalment schedule, compute the equivalent instalment:*

$$C^{q*}(m) = \frac{1}{I^*(m)} \sum_{i=1}^{I} C_i^q(m).$$

3. *Let S be the total number of simulations and* $sd(t)$ *the number of surviving debtors at time t.*

4. *For each of the S simulations, set:* $sd(0) = I^*(0)$, $C_P(0) = sd(0) \cdot C^{q*}(0)$, $L_P(0) = 0$, $\Pi_s(t) = 0$, $\Pi_p = 0$, $cnd\%(0) = 0$.

[2]See Castagna [3] and the references therein for the same approach applied to a portfolio of receivables.

5. *For each time* $t \in \{1, \ldots, T\}$, *draw the number of defaults* $nd(t)$ *from a binomial distribution* $\mathcal{B}(sd(t-1), \mathbf{PD}^*(t-1,t))$.

6. *Update the cumulative default percentage:*

$$cnd\%(t) = cnd\%(t-1) + \frac{nd(t)}{sd(t-1)},$$

if $sd(t-1) = 0$ *then* $cnd\%(t) = 1$.

7. *Update the total credit loss:*

$$L_P(t) = L_P(t-1) + nd(t) \cdot \mathbf{Lgd} \cdot \mathbf{EAD}^*(t-1).$$

8. *Update the posted collateral:*

$$C_P(t) = C_P(t) + [sd(t-1) - nd(t)] \cdot C^{q*}(t).$$

9. *Update the number of surviving debtors:*

$$sd(t) = \lfloor I^*(t) \cdot [1 - cnd\%(t)] \rfloor .$$

10. *Update the premium:*

$$\Pi_s(t) = \Pi_s(t-1) + P(0,t)\left[\min(L_P(t), C_P(t)) - \min(L_P(t-1), C_P(t-1))\right].$$

11. *Store the premium:*

$$\Pi_p = \Pi_p + \Pi_s(T).$$

12. *After completing all S simulations, compute the final premium:*

$$\Pi = \frac{\Pi_p}{S}.$$

Procedure 4.1 is quite efficient and easily implemented.

5. Application to a Realistic Portfolio

We apply the framework sketched above to a realistic portfolio consisting of 100 loans. The details of each loan are shown in Appendix A.1. Table 6 provides, for each debtor, the initial amount and the amortisation schedule. All loans start at time $t = 0$ and mature at time $t = 5$.
In Table 7, we report the probability of default **PD** for each debtor under two different scenarios: the first with moderately low probabilities, and the second with higher ones. We assume that **PD** is constant over each of the five years of the loan duration. The (weighted) average **PD** for the entire portfolio is 3.56% in the first scenario and 5.40% in the second.
For each probability scenario, we also consider the expected collateral contributed by each debtor under two assumptions:

- A single contribution made fully at the inception of the loan;
- A contribution made in five instalments, one at the beginning of each year.

In the first case, the possibility of default has no impact on the contributed amount. In both cases, the total contributed collateral (if the debtor does not default before completing the instalment schedule) is 10% of the initial borrowed amount.
Given the expected contributed collateral $EC_i(5)$, we compute the parameters $\beta_i = \alpha_i$ that make the expected return in Equation (12) equal to zero for each debtor. The total collateral for each probability of default scenario and contribution schedule is reported in Table 1. It is clear that the

EC_p		
1 Inst.	**5 Inst. Scen. 1**	**5 Inst. Scen. 2**
€ 28,120	€ 26,188	€ 25,269

TABLE 1: Total collateral contributed under different probability of default scenarios and collateral contribution schedules.

	Year					
	0	**1**	**2**	**3**	**4**	**5**
Average Loss		€ 6,014	€ 4,881	€ 3,715	€ 2,513	€ 1,275
Loss M^2		€ 12,068,171	€ 7,950,128	€ 4,603,759	€ 2,106,732	€ 542,365
Loss Variance		€ 3,474	€ 2,820	€ 2,146	€ 1,451	€ 736
PD*		3.56%	3.56%	3.56%	3.56%	3.56%
EAD*	€ 3,468	€ 2,815	€ 2,142	€ 1,449	€ 735	€ -
I*	100	81	81	81	81	81

TABLE 2: Approximation by MMT to a homogeneous portfolio in the first scenario for probability of default.

smaller expected amount of collateral (25,269) refers to an accumulation plan in 5 instalments in the higher probability of deafult scenario.
We now compute the total premium Π, which should be paid by the lender to the debtors, under two different probability of default scenarios and five assumptions regarding the collateral contribution schedule, ranging from 1 to 5 instalments.
For the computation, we first apply the Moment Matching Technique (MMT) described in Section 4 to approximate the actual portfolio with an equivalent homogeneous portfolio in each of the five years. For the first **PD** scenario, the expected loss, second moment, and variance-along with the three quantities solving the system in Equation (24) - are reported in Table 2. The same results for the second **PD** scenario are shown in Table 3.
We now implement Procedure 4.1 to compute the total premium Π, which should be paid by the lender to the debtor companies. The computation is performed under the two different **PD** scenarios and across five assumptions regarding the collateral contribution schedule, ranging from 1 to 5 instalments.
The results for the total premium Π in absolute terms are reported in Table 4, while the same results expressed as a percentage of the initial amount of the loan portfolio are shown in Table 5.
It can be noted that the level of the premium is, not surprisingly, linked to the average level of the default probabilities of the debtors. In the first case (average of 3.56% p.a.), the premium is around 5.70% of the initial notional of the loans, for a total theoretical collateral pool of 10% of the same value. In the second case (5.40% p.a.), the premium varies from around 8.00% to 7.50%.
More unexpectedly, the dependence of the premium on the collateral contribution schedule is very mild: it is practically negligible in the first **PD** scenario, and around 0.5% in the second scenario.
As a final application, we show how to calculate the total premium including the extra payment Φ that ensures financial fairness. Consider debtor 1, with all details taken from the tables in Appendix A.1, for the first **PD** scenario. We assume that both the fraction of the premium $\alpha_1\Pi$ and the extra payment Φ_1 are paid at the start of each year of the loan's duration, regardless of whether the debtor survives until expiry or defaults earlier.
Let us assume that the annual risk-free interest rate is constant at 3.5%. The discount factors are given by:

$$P(0, n) = \exp(-0.035 \times n).$$

	Year					
	0	1	2	3	4	5
Average Loss		€ 9,103	€ 7,388	€ 5,622	€ 3,803	€ 1,930
Loss M²		€ 17,895,414	€ 11,788,931	€ 6,826,732	€ 3,123,989	€ 804,251
Loss Variance		€ 4,230	€ 3,434	€ 2,613	€ 1,767	€ 897
PD*		5.40%	5.40%	5.40%	5.40%	5.40%
EAD*	€ 3,463	€ 2,811	€ 2,139	€ 1,447	€ 734	€ -
I*	100	81	81	81	81	81

TABLE 3: Approximation by MMT to a homogeneous portfolio in the second scenario for probability of default.

	Installments				
PD Scen.	1	2	3	4	5
3.56%	€ 16,129	€ 16,067	€ 16,031	€ 16,011	€ 15,970
5.40%	€ 22,691	€ 22,354	€ 21,981	€ 21,624	€ 21,128

TABLE 4: Premium Π for the credit protection, in each of the two scenarios of default probabilities and collateral contribution schedule with different number of instalments. Absolute values.

Additionally, we assume that the collateral is contributed by debtor 1 in 5 instalments at the beginning of each year of the loan's duration. The probability of default is taken from the first scenario.
First, we compute the quantity ξ in Equation (18), and we have:

$$\xi = \frac{\alpha_i^* \Pi}{\alpha_i^* \Pi \sum_{t=0}^{T} P(0,t)}.$$

Since $\alpha_1 \Pi = 242.18$, we get:

$$\xi = 0.21.$$

Calculation of the extra payment Φ_i is now straightforward from Equation (19):

$$\Phi_i = -51.60.$$

which is negative, as expected.

	Installments				
PD Scen.	1	2	3	4	5
3.56%	5.74%	5.71%	5.70%	5.69%	5.68%
5.40%	8.07%	7.95%	7.82%	7.69%	7.51%

TABLE 5: *Premium Π for the credit protection, in each of the two scenarios of default probabilities and collateral contribution schedule with different number of instalments. In percentage of the initial value of the portfolio of loans.*

6. Conclusion

We proposed a collateral pooling scheme that offers credit protection to the lender of a portfolio of loans to corporate debtors. The framework is flexible enough to allow for the contribution of collateral in instalments, even during the life of the loans.

The value of the credit protection offered by the collateral, and the corresponding premium to be paid to the debtors, has been derived. The allocation of this premium and of the remaining collateral at the expiry of the loan portfolio is determined based on actuarial and financial fairness conditions. More sophisticated schemes and alternative options for allocating the residual collateral - and consequently the premium - can be designed starting from the present framework. Also a credit model that allows for a correlation of default events would be make the framework more effective.

References

[1] **Abdikerimova, S. and Feng, R.** *Peer-to-peer multi-risk insurance and mutual aid*. European Journal of Operational Research, Vol. 299, Issue 2, pp. 735-749, June 2022.

[2] **Brigo, D., Pallavicini, A. and Torresetti, R.** *Calibration of CDO Tranches with the Dynamical Generalized-Poisson Loss Model*. SSRN Electronic Journal, May 2007.

[3] **Castagna, A.** *Risk Sharing Insurance Schemes for Invoice Discounting Platforms*. Research Paper Series N. 42. February 2022.

[4] **Denuit, M. and Dhaeneb, J.** *Convex order and comonotonic conditional mean risk sharing*. Insurance: Mathematics and Economics, Vol. 51, Issue 2, pp. 265-270, September 2012.

[5] **Denuit, M. and Robert, C.** *Risk sharing under the dominant peerâtoâpeer property and casualty insurance business models*. Risk Management and Insurance Review, Vol. 24, Issue 2, pp. 181-205, June 2021.

[6] **Duffie, D.** *First-to-Default Valuation*. Stanford University, May 1998.

[7] **Duffie, D. and Garleanu, N.** *Risk and Valuation of Collateralized Debt Obligations*. Stanford University, September 2001.

[8] **Duffie, D. and Singleton, K.** *Simulating Correlated Defaults*. Stanford University, May 1999.

[9] **Feng, R., Liu, C. and Taylor, S.** *Peer-to-Peer Risk Sharing with an Application to Flood Risk Pooling*. Annals of Operations Research, Vol. 321, pp. 813-842, July 2022.

A. *Annex*

A.1 Loan Portfolio Details

Table shows the details of the portfolio of loans. Each loan starts on the same date $t = 0$ and expires on $t = 5$.

	Notional (K(0)) at Year	Amortisation (A(t)) at Year				
ID Debtor	**0**	**1**	**2**	**3**	**4**	**5**
1	€ 4,200	€ 791	€ 815	€ 839	€ 864	€ 890
2	€ 4,500	€ 848	€ 873	€ 899	€ 926	€ 954
3	€ 3,500	€ 659	€ 679	€ 699	€ 720	€ 742
4	€ 4,700	€ 885	€ 912	€ 939	€ 967	€ 996
5	€ 3,700	€ 697	€ 718	€ 739	€ 762	€ 784
6	€ 700	€ 132	€ 136	€ 140	€ 144	€ 148
7	€ 700	€ 132	€ 136	€ 140	€ 144	€ 148
8	€ 2,900	€ 546	€ 563	€ 579	€ 597	€ 615
9	€ 2,700	€ 509	€ 524	€ 540	€ 556	€ 572
10	€ 1,000	€ 188	€ 194	€ 200	€ 206	€ 212
11	€ 2,000	€ 377	€ 388	€ 400	€ 412	€ 424
12	€ 4,900	€ 923	€ 951	€ 979	€ 1,009	€ 1,039
13	€ 3,000	€ 565	€ 582	€ 599	€ 617	€ 636
14	€ 3,900	€ 735	€ 757	€ 779	€ 803	€ 827
15	€ 3,300	€ 622	€ 640	€ 659	€ 679	€ 700
16	€ 4,100	€ 772	€ 795	€ 819	€ 844	€ 869
17	€ 2,500	€ 471	€ 485	€ 500	€ 515	€ 530
18	€ 3,400	€ 640	€ 660	€ 679	€ 700	€ 721
19	€ 5,000	€ 942	€ 970	€ 999	€ 1,029	€ 1,060
20	€ 4,800	€ 904	€ 931	€ 959	€ 988	€ 1,018
21	€ 700	€ 132	€ 136	€ 140	€ 144	€ 148
22	€ 4,300	€ 810	€ 834	€ 859	€ 885	€ 912
23	€ 4,600	€ 866	€ 892	€ 919	€ 947	€ 975
24	€ 2,900	€ 546	€ 563	€ 579	€ 597	€ 615
25	€ 1,200	€ 226	€ 233	€ 240	€ 247	€ 254
26	€ 1,600	€ 301	€ 310	€ 320	€ 329	€ 339
27	€ 2,700	€ 509	€ 524	€ 540	€ 556	€ 572
28	€ 4,400	€ 829	€ 854	€ 879	€ 906	€ 933
29	€ 800	€ 151	€ 155	€ 160	€ 165	€ 170
30	€ 1,200	€ 226	€ 233	€ 240	€ 247	€ 254

TABLE 6: Premium Π for the credit protection, in each of the two scenarios of default probabilities and collateral contribution schedule with different number of instalments. In percentage of the initial value of the portfolio of loans.

	Notional (K(0)) at Year	Amortisation (A(t)) at Year				
ID Debtor	0	1	2	3	4	5
31	€ 2,100	€ 396	€ 407	€ 420	€ 432	€ 445
32	€ 3,500	€ 659	€ 679	€ 699	€ 720	€ 742
33	€ 2,300	€ 433	€ 446	€ 460	€ 473	€ 488
34	€ 2,400	€ 452	€ 466	€ 480	€ 494	€ 509
35	€ 4,700	€ 885	€ 912	€ 939	€ 967	€ 996
36	€ 3,900	€ 735	€ 757	€ 779	€ 803	€ 827
37	€ 700	€ 132	€ 136	€ 140	€ 144	€ 148
38	€ 1,700	€ 320	€ 330	€ 340	€ 350	€ 360
39	€ 2,600	€ 490	€ 504	€ 520	€ 535	€ 551
40	€ 2,500	€ 471	€ 485	€ 500	€ 515	€ 530
41	€ 4,500	€ 848	€ 873	€ 899	€ 926	€ 954
42	€ 1,000	€ 188	€ 194	€ 200	€ 206	€ 212
43	€ 4,300	€ 810	€ 834	€ 859	€ 885	€ 912
44	€ 700	€ 132	€ 136	€ 140	€ 144	€ 148
45	€ 4,500	€ 848	€ 873	€ 899	€ 926	€ 954
46	€ 2,500	€ 471	€ 485	€ 500	€ 515	€ 530
47	€ 3,000	€ 565	€ 582	€ 599	€ 617	€ 636
48	€ 3,700	€ 697	€ 718	€ 739	€ 762	€ 784
49	€ 3,500	€ 659	€ 679	€ 699	€ 720	€ 742
50	€ 4,700	€ 885	€ 912	€ 939	€ 967	€ 996
51	€ 2,200	€ 414	€ 427	€ 440	€ 453	€ 466
52	€ 3,500	€ 659	€ 679	€ 699	€ 720	€ 742
53	€ 3,400	€ 640	€ 660	€ 679	€ 700	€ 721
54	€ 1,900	€ 358	€ 369	€ 380	€ 391	€ 403
55	€ 1,000	€ 188	€ 194	€ 200	€ 206	€ 212
56	€ 2,600	€ 490	€ 504	€ 520	€ 535	€ 551
57	€ 600	€ 113	€ 116	€ 120	€ 123	€ 127
58	€ 3,600	€ 678	€ 698	€ 719	€ 741	€ 763
59	€ 3,200	€ 603	€ 621	€ 639	€ 659	€ 678
60	€ 4,600	€ 866	€ 892	€ 919	€ 947	€ 975
61	€ 3,600	€ 678	€ 698	€ 719	€ 741	€ 763
62	€ 4,000	€ 753	€ 776	€ 799	€ 823	€ 848
63	€ 700	€ 132	€ 136	€ 140	€ 144	€ 148
64	€ 4,700	€ 885	€ 912	€ 939	€ 967	€ 996
65	€ 700	€ 132	€ 136	€ 140	€ 144	€ 148

TABLE 6: *Premium Π for the credit protection, in each of the two scenarios of default probabilities and collateral contribution schedule with different number of instalments. In percentage of the initial value of the portfolio of loans.*

	Notional (K(0)) at Year	Amortisation (A(t)) at Year				
ID Debtor	0	1	2	3	4	5
66	€ 4,000	€ 753	€ 776	€ 799	€ 823	€ 848
67	€ 4,900	€ 923	€ 951	€ 979	€ 1,009	€ 1,039
68	€ 600	€ 113	€ 116	€ 120	€ 123	€ 127
69	€ 2,000	€ 377	€ 388	€ 400	€ 412	€ 424
70	€ 3,500	€ 659	€ 679	€ 699	€ 720	€ 742
72	€ 2,300	€ 433	€ 446	€ 460	€ 473	€ 488
73	€ 2,600	€ 490	€ 504	€ 520	€ 535	€ 551
74	€ 4,500	€ 848	€ 873	€ 899	€ 926	€ 954
75	€ 1,500	€ 283	€ 291	€ 300	€ 309	€ 318
76	€ 900	€ 170	€ 175	€ 180	€ 185	€ 191
77	€ 3,700	€ 697	€ 718	€ 739	€ 762	€ 784
78	€ 2,800	€ 527	€ 543	€ 560	€ 576	€ 594
79	€ 3,500	€ 659	€ 679	€ 699	€ 720	€ 742
80	€ 800	€ 151	€ 155	€ 160	€ 165	€ 170
81	€ 4,500	€ 848	€ 873	€ 899	€ 926	€ 954
82	€ 500	€ 94	€ 97	€ 100	€ 103	€ 106
83	€ 3,900	€ 735	€ 757	€ 779	€ 803	€ 827
84	€ 1,000	€ 188	€ 194	€ 200	€ 206	€ 212
85	€ 1,000	€ 188	€ 194	€ 200	€ 206	€ 212
86	€ 1,700	€ 320	€ 330	€ 340	€ 350	€ 360
87	€ 2,700	€ 509	€ 524	€ 540	€ 556	€ 572
88	€ 3,900	€ 735	€ 757	€ 779	€ 803	€ 827
89	€ 4,500	€ 848	€ 873	€ 899	€ 926	€ 954
90	€ 3,200	€ 603	€ 621	€ 639	€ 659	€ 678
91	€ 3,700	€ 697	€ 718	€ 739	€ 762	€ 784
92	€ 2,500	€ 471	€ 485	€ 500	€ 515	€ 530
93	€ 3,700	€ 697	€ 718	€ 739	€ 762	€ 784
94	€ 1,300	€ 245	€ 252	€ 260	€ 268	€ 276
95	€ 1,300	€ 245	€ 252	€ 260	€ 268	€ 276
96	€ 2,100	€ 396	€ 407	€ 420	€ 432	€ 445
97	€ 4,700	€ 885	€ 912	€ 939	€ 967	€ 996
98	€ 3,600	€ 678	€ 698	€ 719	€ 741	€ 763
99	€ 1,000	€ 188	€ 194	€ 200	€ 206	€ 212
100	€ 2,500	€ 471	€ 485	€ 500	€ 515	€ 530

TABLE 6: Premium Π for the credit protection, in each of the two scenarios of default probabilities and collateral contribution schedule with different number of instalments. In percentage of the initial value of the portfolio of loans.

	PD		EC_i			$\beta_i = \alpha_i$		
	Sc.1	Sc.2	1 Inst.	5 Inst. Sc.1	5 Inst. Sc.2	1 Inst.	5 Inst. Sc.1	5 Inst. Sc.2
1	2.80%	7.40%	€ 420	€ 397	€ 362	1.49%	1.52%	1.43%
2	3.30%	7.40%	€ 450	€ 421	€ 388	1.60%	1.61%	1.54%
3	2.50%	1.90%	€ 350	€ 333	€ 337	1.24%	1.27%	1.33%
4	3.30%	8.80%	€ 470	€ 440	€ 394	1.67%	1.68%	1.56%
5	4.40%	2.90%	€ 370	€ 339	€ 349	1.32%	1.29%	1.38%
6	4.70%	8.90%	€ 70	€ 64	€ 59	0.25%	0.24%	0.23%
7	3.20%	4.90%	€ 70	€ 66	€ 63	0.25%	0.25%	0.25%
8	5.00%	3.60%	€ 290	€ 262	€ 270	1.03%	1.00%	1.07%
9	2.60%	7.20%	€ 270	€ 256	€ 234	0.96%	0.98%	0.93%
10	3.50%	4.50%	€ 100	€ 93	€ 91	0.36%	0.36%	0.36%
11	2.70%	7.30%	€ 200	€ 189	€ 173	0.71%	0.72%	0.68%
12	5.00%	3.20%	€ 490	€ 443	€ 460	1.74%	1.69%	1.82%
13	2.50%	7.60%	€ 300	€ 285	€ 258	1.07%	1.09%	1.02%
14	3.40%	8.20%	€ 390	€ 364	€ 331	1.39%	1.39%	1.31%
15	3.00%	1.10%	€ 330	€ 311	€ 323	1.17%	1.19%	1.28%
16	2.90%	4.70%	€ 410	€ 387	€ 373	1.46%	1.48%	1.48%
17	5.00%	6.10%	€ 250	€ 226	€ 221	0.89%	0.86%	0.88%
18	2.90%	6.60%	€ 340	€ 321	€ 298	1.21%	1.23%	1.18%
19	4.50%	3.40%	€ 500	€ 457	€ 467	1.78%	1.74%	1.85%
20	3.20%	4.40%	€ 480	€ 450	€ 440	1.71%	1.72%	1.74%
21	4.70%	8.60%	€ 70	€ 64	€ 59	0.25%	0.24%	0.23%
22	3.30%	5.00%	€ 430	€ 403	€ 389	1.53%	1.54%	1.54%
23	2.60%	4.40%	€ 460	€ 437	€ 421	1.64%	1.67%	1.67%
24	2.60%	2.00%	€ 290	€ 275	€ 279	1.03%	1.05%	1.10%
25	3.40%	1.00%	€ 120	€ 112	€ 118	0.43%	0.43%	0.47%
26	4.20%	5.20%	€ 160	€ 147	€ 144	0.57%	0.56%	0.57%
27	2.90%	7.50%	€ 270	€ 255	€ 232	0.96%	0.97%	0.92%
28	3.60%	5.90%	€ 440	€ 409	€ 391	1.56%	1.56%	1.55%
29	3.60%	7.90%	€ 80	€ 74	€ 68	0.28%	0.28%	0.27%
30	2.90%	1.30%	€ 120	€ 113	€ 117	0.43%	0.43%	0.46%
31	2.60%	6.20%	€ 210	€ 199	€ 186	0.75%	0.76%	0.73%
32	3.10%	3.50%	€ 350	€ 329	€ 326	1.24%	1.26%	1.29%

TABLE 7: Probability of default of each debtor, expected collateral contributed in 1 instalment and 5 instalments and parameters $\beta = \alpha$ for each debtor (in the two different probability of default scearios).

	PD		EC_i			$\beta_i = \alpha_i$		
	Sc.1	**Sc.2**	**1 Inst.**	**5 Inst. Sc.1**	**5 Inst. Sc.2**	**1 Inst.**	**5 Inst. Sc.1**	**5 Inst. Sc.2**
33	3.30%	6.00%	€ 230	€ 215	€ 204	0.82%	0.82%	0.81%
34	3.20%	2.90%	€ 240	€ 225	€ 226	0.85%	0.86%	0.90%
35	3.10%	4.50%	€ 470	€ 442	€ 430	1.67%	1.69%	1.70%
36	4.80%	5.50%	€ 390	€ 354	€ 349	1.39%	1.35%	1.38%
37	3.30%	1.80%	€ 70	€ 66	€ 68	0.25%	0.25%	0.27%
38	3.10%	4.40%	€ 170	€ 160	€ 156	0.60%	0.61%	0.62%
39	3.80%	8.40%	€ 260	€ 241	€ 220	0.92%	0.92%	0.87%
40	4.30%	8.70%	€ 250	€ 229	€ 210	0.89%	0.88%	0.83%
41	2.80%	6.30%	€ 450	€ 425	€ 397	1.60%	1.62%	1.57%
42	4.40%	4.30%	€ 100	€ 92	€ 92	0.36%	0.35%	0.36%
43	3.60%	5.80%	€ 430	€ 400	€ 383	1.53%	1.53%	1.52%
44	4.60%	2.40%	€ 70	€ 64	€ 67	0.25%	0.24%	0.26%
45	3.90%	8.10%	€ 450	€ 416	€ 383	1.60%	1.59%	1.51%
46	4.20%	8.10%	€ 250	€ 230	€ 213	0.89%	0.88%	0.84%
47	3.50%	2.40%	€ 300	€ 280	€ 286	1.07%	1.07%	1.13%
48	4.20%	2.70%	€ 370	€ 340	€ 351	1.32%	1.30%	1.39%
49	2.80%	3.20%	€ 350	€ 331	€ 328	1.24%	1.26%	1.30%
50	4.50%	4.10%	€ 470	€ 430	€ 433	1.67%	1.64%	1.71%
51	2.70%	5.30%	€ 220	€ 208	€ 198	0.78%	0.80%	0.78%
52	3.20%	6.00%	€ 350	€ 328	€ 310	1.24%	1.25%	1.23%
53	2.70%	2.60%	€ 340	€ 322	€ 323	1.21%	1.23%	1.28%
54	4.60%	2.70%	€ 190	€ 173	€ 180	0.68%	0.66%	0.71%
55	3.20%	5.10%	€ 100	€ 94	€ 90	0.36%	0.36%	0.36%
56	3.30%	8.80%	€ 260	€ 243	€ 218	0.92%	0.93%	0.86%
57	3.50%	4.90%	€ 60	€ 56	€ 54	0.21%	0.21%	0.22%
58	3.50%	9.00%	€ 360	€ 336	€ 301	1.28%	1.28%	1.19%
59	2.60%	3.10%	€ 320	€ 304	€ 301	1.14%	1.16%	1.19%
60	3.60%	7.90%	€ 460	€ 428	€ 393	1.64%	1.63%	1.55%
61	3.50%	5.30%	€ 360	€ 336	€ 324	1.28%	1.28%	1.28%
62	3.50%	8.70%	€ 400	€ 373	€ 336	1.42%	1.42%	1.33%
63	4.90%	5.70%	€ 70	€ 63	€ 62	0.25%	0.24%	0.25%
64	3.60%	8.20%	€ 470	€ 437	€ 399	1.67%	1.67%	1.58%

TABLE 7: Probability of default of each debtor, expected collateral contributed in 1 instalment and 5 instalments and parameters $\beta = \alpha$ for each debtor (in the two different probability of default scearios).

	PD		EC_i			$\beta_i = \alpha_i$		
	Sc.1	Sc.2	1 Inst.	5 Inst. Sc.1	5 Inst. Sc.2	1 Inst.	5 Inst. Sc.1	5 Inst. Sc.2
65	3.50%	8.50%	€ 70	€ 65	€ 59	0.25%	0.25%	0.23%
66	4.40%	6.80%	€ 400	€ 366	€ 349	1.42%	1.40%	1.38%
67	3.70%	3.10%	€ 490	€ 455	€ 461	1.74%	1.74%	1.82%
68	2.80%	6.30%	€ 60	€ 57	€ 53	0.21%	0.22%	0.21%
69	3.30%	1.90%	€ 200	€ 187	€ 193	0.71%	0.71%	0.76%
70	3.90%	8.80%	€ 350	€ 324	€ 294	1.24%	1.24%	1.16%
71	3.90%	7.30%	€ 130	€ 120	€ 112	0.46%	0.46%	0.44%
72	3.80%	4.30%	€ 230	€ 213	€ 211	0.82%	0.81%	0.84%
73	2.80%	5.50%	€ 260	€ 246	€ 233	0.92%	0.94%	0.92%
74	4.40%	7.10%	€ 450	€ 412	€ 390	1.60%	1.57%	1.55%
75	4.30%	5.50%	€ 150	€ 138	€ 134	0.53%	0.53%	0.53%
76	4.10%	3.60%	€ 90	€ 83	€ 84	0.32%	0.32%	0.33%
77	3.20%	5.70%	€ 370	€ 347	€ 330	1.32%	1.33%	1.31%
78	4.40%	4.20%	€ 280	€ 256	€ 257	1.00%	0.98%	1.02%
79	3.00%	6.70%	€ 350	€ 330	€ 306	1.24%	1.26%	1.21%
80	4.10%	6.80%	€ 80	€ 74	€ 70	0.28%	0.28%	0.28%
81	4.50%	2.00%	€ 450	€ 411	€ 432	1.60%	1.57%	1.71%
82	4.90%	1.30%	€ 50	€ 45	€ 49	0.18%	0.17%	0.19%
83	4.10%	6.80%	€ 390	€ 359	€ 340	1.39%	1.37%	1.35%
84	2.70%	7.30%	€ 100	€ 95	€ 86	0.36%	0.36%	0.34%
85	3.40%	2.40%	€ 100	€ 93	€ 95	0.36%	0.36%	0.38%
86	4.70%	6.70%	€ 170	€ 155	€ 149	0.60%	0.59%	0.59%
87	4.80%	7.20%	€ 270	€ 245	€ 234	0.96%	0.94%	0.93%
88	4.10%	6.00%	€ 390	€ 359	€ 346	1.39%	1.37%	1.37%
89	2.90%	4.10%	€ 450	€ 425	€ 415	1.60%	1.62%	1.64%
90	3.10%	1.60%	€ 320	€ 301	€ 310	1.14%	1.15%	1.23%
91	4.50%	7.40%	€ 370	€ 338	€ 319	1.32%	1.29%	1.26%
92	4.00%	3.50%	€ 250	€ 231	€ 233	0.89%	0.88%	0.92%
93	2.60%	5.70%	€ 370	€ 351	€ 330	1.32%	1.34%	1.31%
94	3.00%	7.10%	€ 130	€ 122	€ 113	0.46%	0.47%	0.45%
95	4.40%	7.20%	€ 130	€ 119	€ 113	0.46%	0.45%	0.45%
96	4.60%	6.80%	€ 210	€ 192	€ 183	0.75%	0.73%	0.73%
97	2.60%	5.40%	€ 470	€ 446	€ 422	1.67%	1.70%	1.67%
98	4.00%	2.80%	€ 360	€ 332	€ 340	1.28%	1.27%	1.35%
99	4.30%	2.80%	€ 100	€ 92	€ 95	0.36%	0.35%	0.37%
100	2.70%	5.70%	€ 250	€ 237	€ 223	0.89%	0.90%	0.88%

TABLE 7: Probability of default of each debtor, expected collateral contributed in 1 instalment and 5 instalments and parameters $\beta = \alpha$ for each debtor (in the two different probability of default scearios).

Iason is an international firm that consults Financial Institutions on Risk Management. Iason is a leader in quantitative analysis and advanced risk methodology, offering a unique mix of know-how and expertise on the pricing of complex financial products and the management of financial, credit and liquidity risks. In addition Iason provides a suite of essential solutions to meet the fundamental needs of Financial Institutions.